Contents

1 Introduction 1
1.1 Ontology: What There Is 1
1.1.1 Extracts 1
1.1.2 In Brief: The Problem of Quantum Mechanics 4
1.1.3 In Brief: Bohmian Mechanics 6
1.2 Determinism and Realism 9
References 10

2 Classical Physics 11
2.1 Newtonian Mechanics 12
2.2 Hamiltonian Mechanics 13
2.3 Hamilton–Jacobi Formulation 24
2.4 Fields and Particles: Electromagnetism 26
2.5 No fields, Only Particles: Electromagnetism 34
2.6 On the Symplectic Structure of the Phase Space 38
References 42

3 Symmetry 43

4 Chance 49
4.1 Typicality 51
4.1.1 Typical Behavior. The Law of Large Numbers 54
4.1.2 Statistical Hypothesis and Its Justification 63
4.1.3 Typicality in Subsystems: Microcanonical and Canonical Ensembles 66
4.2 Irreversibility 80
4.2.1 Typicality Within Atypicality 81
4.2.2 Our Atypical Universe 89
4.2.3 Ergodicity and Mixing 90
4.3 Probability Theory 96
4.3.1 Lebesgue Measure and Coarse-Graining 96

4.3.2 The Law of Large Numbers . . . 102
References . . . 107

5 Brownian motion . . . 109
5.1 Einstein's Argument . . . 110
5.2 On Smoluchowski's Microscopic Derivation . . . 114
5.3 Path Integration . . . 118
References . . . 119

6 The Beginning of Quantum Theory . . . 121
References . . . 127

7 Schrödinger's Equation . . . 129
7.1 The Equation . . . 129
7.2 What Physics Must Not Be . . . 135
7.3 Interpretation, Incompleteness, and $\rho = |\psi|^2$ . . . 139
References . . . 143

8 Bohmian Mechanics . . . 145
8.1 Derivation of Bohmian Mechanics . . . 147
8.2 Bohmian Mechanics and Typicality . . . 151
8.3 Electron Trajectories . . . 153
8.4 Spin . . . 158
8.5 A Topological View of Indistinguishable Particles . . . 166
References . . . 171

9 The Macroscopic World . . . 173
9.1 Pointer Positions . . . 173
9.2 Effective Collapse . . . 179
9.3 Centered Wave packets . . . 183
9.4 The Classical Limit of Bohmian Mechanics . . . 186
9.5 Some Further Observations . . . 191
9.5.1 Dirac Formalism, Density Matrix, Reduced Density Matrix, and Decoherence . . . 191
9.5.2 Poincaré Recurrence . . . 198
References . . . 200

10 Nonlocality . . . 201
10.1 Singlet State and Probabilities for Anti-Correlations . . . 205
10.2 Faster Than Light Signals? . . . 208
References . . . 209

11 The Wave Function and Quantum Equilibrium . . . 211
11.1 Measure of Typicality . . . 211
11.2 Conditional Wave Function . . . 213
11.3 Effective Wave function . . . 216

11.4 Typical Empirical Distributions . . . 218
11.5 Misunderstandings . . . 223
11.6 Quantum Nonequilibrium . . . 224
References . . . 225

12 From Physics to Mathematics . . . 227
12.1 Observables. An Unhelpful Notion . . . 227
12.2 Who Is Afraid of PVMs and POVMs? . . . 233
12.2.1 The Theory Decides What Is Measurable . . . 241
12.2.2 Joint Probabilities . . . 242
12.2.3 Naive Realism about Operators . . . 244
12.3 Schrödinger's Equation Revisited . . . 245
12.4 What Comes Next? . . . 248
References . . . 249

13 Hilbert Space . . . 251
13.1 The Hilbert Space L^2 . . . 253
13.1.1 The Coordinate Space ℓ^2 . . . 255
13.1.2 Fourier Transformation on L^2 . . . 258
13.2 Bilinear Forms and Bounded Linear Operators . . . 268
13.3 Tensor Product Spaces . . . 271
References . . . 278

14 The Schrödinger Operator . . . 279
14.1 Unitary Groups and Their Generators . . . 279
14.2 Self-Adjoint Operators . . . 284
14.3 The Atomistic Schrödinger Operator . . . 294
References . . . 298

15 Measures and Operators . . . 299
15.1 Examples of PVMs and Their Operators . . . 303
15.1.1 Heisenberg Operators . . . 305
15.1.2 Asymptotic Velocity and the Momentum Operator . . . 306
15.2 The Spectral Theorem . . . 311
15.2.1 The Dirac Formalism . . . 311
15.2.2 Mathematics of the Spectral Theorem . . . 313
15.2.3 Spectral Representations . . . 322
15.2.4 Unbounded Operators . . . 324
15.2.5 Unitary Groups . . . 332
15.2.6 $H_0 = -\Delta/2$ . . . 333
15.2.7 The Spectrum . . . 341
References . . . 344

16 Bohmian Mechanics on Scattering Theory 345
16.1 Exit Statistics 346
16.2 Asymptotic Exits 353
16.3 Scattering Theory and Exit Distribution 356
16.4 More on Abstract Scattering Theory 358
16.5 Generalized Eigenfunctions 361
16.6 Towards the Scattering Cross-Section 368
16.7 The Scattering Cross-Section 369
16.7.1 Born's Formula 370
16.7.2 Time-Dependent Scattering 372
References 378

17 Epilogue 379
References 380

Bibliography 381

Index 387