Contents

1 Geometry of State Spaces
A. Uhlmann, B. Crell 1
1.1 Introduction 1
1.2 Geometry of Pure States 4
1.2.1 Norm and Distance in Hilbert Space 4
1.2.2 Length of Curves in $\mathcal{H}$ 4
1.2.3 Distance and Length 6
1.2.4 Curves on the Unit Sphere 7
1.2.5 Phases 9
1.2.6 Fubini–Study Distance 10
1.2.7 Fubini–Study Metric 12
1.2.8 Symmetries 13
1.2.9 Comparison with Other Norms 15
1.3 Operators, Observables, and States 16
1.3.1 States and Expectation Values 17
1.3.2 Subalgebras and Subsystems 19
1.3.3 Classification of Finite Quantum Systems 23
1.3.4 All Subsystems for $\dim \mathcal{H} < \infty$ 32
1.4 Transition Probability, Fidelity, and Bures Distance 35
1.4.1 Purification 36
1.4.2 Transition Probability and Fidelity 37
1.4.3 Optimization 38
1.4.4 Why the Bures Distance Is a Distance 40
1.4.5 Expressions for Fidelity and Transition Probability 43
1.4.6 Estimates and a "Hidden Symmetry" 47
1.4.7 "Operational Fidelity" 49
1.5 Appendix: The Geometrical Mean 50
1.5.1 Geometric Mean and Fidelity 52
1.5.2 The Transformer Identity 53
1.5.3 #-Convexity 57
References 58

2 Basic Concepts of Entangled States
F. Mintert, C. Viviescas, A. Buchleitner 61
2.1 Introduction 61
2.2 Entangled States 61
2.2.1 Pure States 62
2.2.2 Mixed States 63
2.3 Separability Criteria 64
2.3.1 Pure States 64
2.3.2 Mixed States 66
2.4 Entanglement Monotones and Measures 71
2.4.1 General Considerations 71
2.4.2 Some Specific Monotones and Measures 78
References 86

3 Topology and Quantum Computing
L.H. Kauffman, S.J. Lomonaco Jr. 87
3.1 Introduction 87
3.2 Knots and Braids 90
3.3 Quantum Mechanics and Quantum Computation 94
3.3.1 What Is a Quantum Computer? 95
3.4 Braiding Operators and Universal Quantum Gates 96
3.4.1 Universal Gates 99
3.5 A Remark About EPR, Entanglement and Bell's Inequality 102
3.6 The Aravind Hypothesis 104
3.7 $SU(2)$ Representations of the Artin Braid Group 104
3.8 The Bracket Polynomial and the Jones Polynomial 110
3.8.1 The State Summation 113
3.8.2 Quantum Computation of the Jones Polynomial 114
3.8.3 The Hadamard Test 117
3.9 Quantum Topology, Cobordism Categories, Temperley–Lieb Algebra and Topological Quantum Field Theory 118
3.10 Braiding and Topological Quantum Field Theory 127
3.11 Spin Networks and Temperley–Lieb Recoupling Theory 135
3.11.1 Evaluations 140
3.11.2 Symmetry and Unitarity 141
3.12 Fibonacci Particles 145
3.13 The Fibonacci Recoupling Model 148
3.14 Quantum Computation of Colored Jones Polynomials and the Witten–Reshetikhin–Turaev Invariant 150
References 153

4 Entanglement in Phase Space
A.M. Ozorio de Almeida 157
4.1 Introduction 157
4.2 Entanglement and Classical Physics 160

4.3 Classical–Quantum Correspondence . . . 163
4.4 Semiclassical Quantum States . . . 169
4.5 Operator Representations and Double Phase Space . . . 175
4.6 The Wigner Function and the Chord Function . . . 182
4.7 The Partial Trace: Sections and Projections . . . 193
4.8 Generating a *Classical* Entanglement: The EPR State . . . 200
4.9 Entanglement and Decoherence . . . 206
4.10 A Semiclassical Picture of Entanglement . . . 208
References . . . 215

5 Introduction to Decoherence Theory
K. Hornberger . . . 221
5.1 The Concept of Decoherence . . . 221
5.1.1 Decoherence in a Nutshell . . . 222
5.1.2 General Scattering Interaction . . . 224
5.1.3 Decoherence as an Environmental Monitoring Process . . . 225
5.1.4 A Few Words on Nomenclature . . . 229
5.2 Case Study: Dephasing of Qubits . . . 230
5.2.1 An Exactly Solvable Model . . . 230
5.2.2 The Continuum Limit . . . 234
5.2.3 Dephasing of N Qubits . . . 237
5.3 Markovian Dynamics of Open Quantum Systems . . . 239
5.3.1 Quantum Dynamical Semigroups . . . 240
5.3.2 The Lindblad Form . . . 242
5.3.3 Quantum Trajectories . . . 245
5.3.4 Exemplary Master Equations . . . 249
5.4 Microscopic Derivations . . . 255
5.4.1 The Weak Coupling Formulation . . . 255
5.4.2 The Monitoring Approach . . . 259
5.4.3 Collisional Decoherence of a Brownian Particle . . . 262
5.4.4 Decoherence of a Quantum Dot . . . 267
5.5 Robust States and the Pointer Basis . . . 270
5.5.1 Nonlinear Equation for Robust States . . . 271
5.5.2 Applications . . . 273
References . . . 274

6 Diffusive Spin Transport
C.A. Müller . . . 277
6.1 Introduction . . . 277
6.2 Spin Relaxation . . . 278
6.2.1 Spin – A Primer on Rotations . . . 279
6.2.2 Master Equation Approach to Spin Relaxation . . . 282
6.2.3 Irreducible Scalar Spin Relaxation Rates . . . 287
6.3 Diffusion . . . 294
6.3.1 Transport . . . 294

6.3.2 Momentum – a Primer on Translations 295
6.3.3 Master Equation Approach to Diffusion 296
6.3.4 Linear Response and Diffusion Constant 303
6.4 Diffusive Spin Transport 306
6.4.1 Master Equation Approach to Diffusive Spin Transport . . . 306
6.4.2 Quantum Corrections 309
References 312

Index 315