

Contents

1 Conjugate Direction Methods for Quadratic Problems	1
1.1 Introduction	1
1.2 Conjugate Direction Methods	4
1.3 Conjugate Gradient Error Algorithms	13
1.4 Conjugate Gradient Residual Algorithms	21
1.5 The Method of Shortest Residuals	26
1.6 Rate of Convergence	28
1.7 Relations with a Direct Solver for a Set of Linear Equations	38
1.8 Limited Memory Quasi-Newton Algorithms	45
1.9 Reduced-Hessian Quasi-Newton Algorithms	53
1.10 Limited Memory Reduced-Hessian Quasi-Newton Algorithms	56
1.11 Conjugate Non-Gradient Algorithm	59
1.12 Notes	61
2 Conjugate Gradient Methods for Nonconvex Problems	63
2.1 Introduction	63
2.2 Line Search Methods	65
2.3 General Convergence Results	70
2.4 Moré–Thuente Step-Length Selection Algorithm	76
2.5 Hager–Zhang Step-Length Selection Algorithm	80
2.6 Nonlinear Conjugate Gradient Algorithms	84
2.7 Global Convergence of the Fletcher–Reeves Algorithm	87
2.8 Other Versions of the Fletcher–Reeves Algorithm	96
2.9 Global Convergence of the Polak–Ribi�re Algorithm	98
2.10 Hestenes–Stiefel Versions of the Standard Conjugate Gradient Algorithm	101
2.11 Notes	106
3 Memoryless Quasi-Newton Methods	109
3.1 Introduction	109

3.2	Conjugate Gradient Algorithm as the Memoryless Quasi-Newton Method	112
3.3	Beale's Restart Rule	116
3.4	Convergence of the Shanno Algorithm	118
3.5	Notes.....	127
4	Preconditioned Conjugate Gradient Algorithms	133
4.1	Introduction	133
4.2	Rates of Convergence	134
4.3	Conjugate Gradient Algorithm and the Newton Method.....	139
4.4	Generic Preconditioned Conjugate Gradient Algorithm	145
4.5	Quasi-Newton-like Variable Storage Conjugate Gradient Algorithm	148
4.6	Scaling the Identity	155
4.7	Notes.....	158
5	Limited Memory Quasi-Newton Algorithms.....	159
5.1	Introduction	159
5.2	Global Convergence of the Limited Memory BFGS Algorithm.....	163
5.3	Compact Representation of the BFGS Approximation of the Inverse Hessian Matrix	169
5.4	The Compact Representation of the BFGS Approximation of the Hessian Matrix.....	175
5.5	Numerical Experiments	178
5.6	Notes.....	186
6	The Method of Shortest Residuals and Nondifferentiable Optimization	191
6.1	Introduction	191
6.2	The Method of Conjugate Subgradient by Lemaréchal and Wolfe ..	192
6.3	Conjugate Subgradient Algorithm for Problems with Semismooth Functions	204
6.4	Subgradient Algorithms with Finite Storage	210
6.5	Notes.....	215
7	The Method of Shortest Residuals for Differentiable Problems	217
7.1	Introduction	217
7.2	General Algorithm	219
7.3	Versions of the Method of Shortest Residuals	226
7.4	Polak–Ribièvre Version of the Method of Shortest Residuals	230
7.5	The Method of Shortest Residuals by Dai and Yuan	233
7.6	Global Convergence of the Lemaréchal–Wolfe Algorithm Without Restarts	241
7.7	A Counter-Example	244
7.8	Numerical Experiments: First Comparisons.....	250

7.9	Numerical Experiments: Comparison to the Memoryless Quasi-Newton Method	256
7.10	Numerical Experiments: Comparison to the Limited Memory Quasi-Newton Method	257
7.11	Numerical Experiments: Comparison to the Hager–Zhang Algorithm	264
7.12	Notes	267
8	The Preconditioned Shortest Residuals Algorithm	279
8.1	Introduction	279
8.2	General Preconditioned Method of Shortest Residuals	281
8.3	Globally Convergent Preconditioned Conjugate Gradient Algorithm	286
8.4	Scaling Matrices	288
8.5	Conjugate Gradient Algorithm with the BFGS Scaling Matrices ..	291
8.6	Numerical Experiments	293
8.7	Notes	297
9	Optimization on a Polyhedron	299
9.1	Introduction	299
9.2	Exposed Sets	308
9.3	The Identification of Active Constraints	310
9.4	Gradient Projection Method	313
9.5	On the Stopping Criterion	319
9.6	Notes	324
10	Conjugate Gradient Algorithms for Problems with Box Constraints ..	327
10.1	Introduction	327
10.2	General Convergence Theory	328
10.3	The Globally Convergent Polak–Ribi��re Version of Algorithm 10.1	342
10.4	Convergence Analysis of the Fletcher–Reeves Version of Algorithm 10.1	350
10.5	Numerical Experiments	357
10.6	Notes	359
11	Preconditioned Conjugate Gradient Algorithms for Problems with Box Constraints	371
11.1	Introduction	371
11.2	Active Constraints Identification by Solving Quadratic Problem ..	371
11.3	The Preconditioned Shortest Residuals Algorithm: Outline	376
11.4	The Preconditioned Shortest Residuals Algorithm: Global Convergence	384
11.5	The Limited Memory Quasi-Newton Method for Problems with Box Constraints	386
11.6	Numerical Algebra	387

11.7 Algorithm 11.1 and Algorithm 11.2 Applied to Problems with Strongly Convex Functions	389
11.8 Numerical Experiments	390
11.9 Notes.....	391
12 Preconditioned Conjugate Gradient Based Reduced-Hessian Methods	399
12.1 Introduction	399
12.2 BFGS Update with Column Scaling	400
12.3 The Preconditioned Method of Shortest Residuals with Column Scaling	409
12.4 Reduced-Hessian Quasi-Newton Algorithms	412
12.5 Limited Memory Reduced-Hessian Quasi-Newton Algorithms.....	420
12.6 Preconditioned Conjugate Gradient Based Reduced-Hessian Algorithm.....	424
12.7 Notes.....	428
A Elements of Topology and Analysis	429
A.1 The Topology of the Euclidean Space.....	429
A.2 Continuity and Convexity	432
A.3 Derivatives	435
B Elements of Linear Algebra	441
B.1 Vector and Matrix Norms	441
B.2 Spectral Decomposition	442
B.3 Determinant and Trace	447
C Elements of Numerical Linear Algebra	449
C.1 Condition Number and Linear Equations	449
C.2 The LU and Cholesky Factorizations	450
C.3 The QR Factorization	452
C.4 Householder QR	454
C.5 Givens QR	456
C.6 Gram–Schmidt QR	459
References.....	463
Index	473