

---

# Contents

<b>16</b>	<b>Free-Electron Model of Metals</b> . . . . .	1
16.1	Classical Drude Model . . . . .	2
16.1.1	Basic Assumptions of the Model . . . . .	2
16.1.2	Electrical Conductivity . . . . .	5
16.1.3	Heat Conduction . . . . .	9
16.1.4	Hall Resistance . . . . .	11
16.1.5	AC Conductivity . . . . .	14
16.1.6	High-Frequency Behavior of a Classical Electron Gas . .	16
16.1.7	Magnetic Properties . . . . .	20
16.1.8	Failures of the Drude Model . . . . .	22
16.2	Quantum Mechanical Sommerfeld Model . . . . .	24
16.2.1	Quantum Mechanical States of Free Electrons . . . . .	24
16.2.2	Ground State of the Electron Gas . . . . .	28
16.2.3	Excited Electron and Hole States . . . . .	31
16.2.4	Density of States of the Electron Gas . . . . .	32
16.2.5	Ideal Electron Gas at Finite Temperatures . . . . .	34
16.2.6	Sommerfeld Expansion . . . . .	37
16.2.7	Specific Heat of the Electron Gas . . . . .	40
16.2.8	Equation of State for the Ideal Electron Gas . . . . .	42
16.2.9	Susceptibility of the Electron Gas . . . . .	44
16.3	Electric and Heat Currents in an Electron Gas . . . . .	47
16.3.1	Noninteracting Electrons in a Uniform Electric Field . . .	47
16.3.2	Stationary Distribution Function . . . . .	49
16.3.3	Electric and Heat Currents . . . . .	53
16.3.4	Thermoelectric Phenomena . . . . .	56
16.3.5	Galvanomagnetic and Thermomagnetic Phenomena . . . .	61
16.4	Scattering of Free Electrons by Impurities . . . . .	64
16.4.1	Formal Solution of the Schrödinger Equation . . . . .	64
16.4.2	Approach Based on Scattering Theory . . . . .	65
16.4.3	Friedel Oscillations Around Impurities . . . . .	69
16.4.4	Bound States Around Impurities . . . . .	73
16.5	Inadequacies of the Free-Electron Model . . . . .	74

<b>17</b>	<b>Electrons in the Periodic Potential of a Crystal</b> . . . . .	77
17.1	Band Structure of Electronic States . . . . .	78
17.1.1	Bloch States . . . . .	78
17.1.2	Energy Levels of Bloch States . . . . .	80
17.1.3	Eigenvalue Problem for Equivalent $\mathbf{k}$ Vectors . . . . .	82
17.1.4	Role of the Spin–Orbit Interaction . . . . .	83
17.2	Representation of the Band Structure . . . . .	84
17.2.1	Reduced-, Repeated-, and Extended-Zone Schemes . . . . .	85
17.2.2	Constant-Energy Surfaces and the Fermi Surface . . . . .	88
17.3	Metals, Insulators, Semiconductors . . . . .	89
17.4	Bloch Electrons as Quasiparticles . . . . .	92
17.4.1	Creation and Annihilation Operators of Bloch States . . . . .	92
17.4.2	Effective Mass of Bloch Electrons . . . . .	93
17.4.3	Bloch Electrons and Holes . . . . .	95
17.4.4	Density of States for Bloch Electrons . . . . .	96
17.4.5	Specific Heat and Susceptibility of Bloch Electrons . . . . .	98
17.5	Wannier States . . . . .	100
17.5.1	Wannier Functions . . . . .	100
17.5.2	Creation and Annihilation Operators of Wannier States . . . . .	103
17.6	Electron States Around Impurities . . . . .	104
<b>18</b>	<b>Simple Models of the Band Structure</b> . . . . .	109
18.1	Nearly-Free-Electron Approximation . . . . .	109
18.1.1	Band Structure in the Empty Lattice . . . . .	110
18.1.2	Fermi Surface in the Empty Lattice . . . . .	115
18.1.3	Effects of a Weak Periodic Potential . . . . .	120
18.1.4	Lifting of Accidental Degeneracies . . . . .	126
18.1.5	Fermi Surface for Nearly Free Electrons . . . . .	136
18.2	Tight-Binding Approximation . . . . .	139
18.2.1	Broadening of Atomic Levels into Bands . . . . .	140
18.2.2	Band of $s$ -Electrons . . . . .	143
18.2.3	Band of $p$ -Electrons . . . . .	145
<b>19</b>	<b>Methods for Calculating and Measuring the Band Structure</b> . . . . .	151
19.1	Matrix Methods . . . . .	152
19.1.1	General Formulation of the Problem . . . . .	152
19.1.2	LCAO Method . . . . .	154
19.1.3	Plane-Wave Method . . . . .	156
19.1.4	Orthogonalized-Plane-Wave Method . . . . .	157
19.1.5	Pseudopotential Method . . . . .	160
19.2	Variational Methods and Methods Based on Scattering Theory . . . . .	164
19.2.1	Augmented-Plane-Wave Method . . . . .	164
19.2.2	Green Function or KKR Method . . . . .	168
19.2.3	Physical Interpretation of the KKR Method . . . . .	173

19.2.4	LMTO Method .....	175
19.3	Band Structure and Fermi Surface of Simple Metals .....	178
19.3.1	Monovalent Metals .....	178
19.3.2	Divalent Metals .....	182
19.3.3	Trivalent Metals .....	183
19.3.4	Tetravalent Elements .....	185
19.3.5	Band Structure of Transition Metals and Rare-Earth Metals .....	185
19.4	Experimental Study of the Band Structure .....	187
19.4.1	Positron Annihilation and Compton Scattering .....	187
19.4.2	Photoelectron Spectroscopy .....	190
<b>20</b>	<b>Electronic Structure of Semiconductors .....</b>	<b>195</b>
20.1	Semiconductor Materials .....	196
20.1.1	Elemental Semiconductors .....	196
20.1.2	Compound Semiconductors .....	198
20.2	Band Structure of Pure Semiconductors .....	201
20.2.1	Electronic Structure in the Diamond Lattice .....	201
20.2.2	Band Structure of Silicon .....	204
20.2.3	Band Structure of Germanium .....	206
20.2.4	Band Structure of Compound Semiconductors .....	208
20.2.5	Indirect- and Direct-Gap Semiconductors .....	211
20.3	Electrons and Holes in Intrinsic Semiconductors .....	212
20.3.1	Number of Thermally Excited Charge Carriers .....	212
20.3.2	Temperature Dependence of the Chemical Potential ...	218
20.4	Electronic Structure of Doped Semiconductors .....	219
20.4.1	Energy of Donor and Acceptor Levels .....	220
20.5	Doped Semiconductors at Finite Temperatures .....	224
20.5.1	Condition of Charge Neutrality .....	225
20.5.2	Thermal Population of Donor and Acceptor Levels.....	227
20.5.3	Number of Carriers in Doped Semiconductors .....	228
<b>21</b>	<b>Semiclassical Dynamics of Electrons .....</b>	<b>239</b>
21.1	Basics of Semiclassical Dynamics .....	239
21.1.1	Velocity of Bloch Electrons .....	240
21.1.2	Semiclassical Equation of Motion .....	243
21.1.3	Effective-Mass Tensor .....	246
21.1.4	Motion of Electrons and Holes .....	247
21.2	Bloch Electrons in Uniform Magnetic Fields .....	248
21.2.1	Motion in Reciprocal and Real Spaces .....	249
21.2.2	Open and Closed Orbits in Magnetic Fields .....	251
21.2.3	Cyclotron Frequency in a Closed Orbit .....	253
21.2.4	Cyclotron Mass .....	255
21.2.5	Cyclotron Resonance .....	259
21.2.6	Azbel–Kaner Resonance .....	262

21.2.7	Magnetoacoustic Geometric Oscillations . . . . .	265
21.3	Size Effects . . . . .	268
21.3.1	Extinction of the Resonance in Thin Samples . . . . .	268
21.3.2	Radiofrequency Size Effect . . . . .	270
21.4	Limitations of the Semiclassical Description . . . . .	271
21.4.1	Conditions of the Applicability of Semiclassical Dynamics . . . . .	272
21.4.2	Electric and Magnetic Breakdown . . . . .	274
<b>22</b>	<b>Electrons in Strong Magnetic Fields . . . . .</b>	<b>277</b>
22.1	Free Electrons in a Magnetic Field . . . . .	277
22.1.1	One-Particle Energy Spectrum . . . . .	277
22.1.2	Degree of Degeneracy of Landau Levels . . . . .	281
22.1.3	Density of States . . . . .	283
22.1.4	Visualization of the Landau States . . . . .	288
22.1.5	Landau States in the Symmetric Gauge . . . . .	290
22.1.6	Edge States . . . . .	294
22.2	Landau Diamagnetism . . . . .	295
22.3	Bloch Electrons in Strong Magnetic Fields . . . . .	297
22.3.1	Electrons Characterized by an Effective-Mass Tensor . . . . .	297
22.3.2	Semiclassical Quantization . . . . .	300
22.3.3	Quantization of the Orbit in Real Space . . . . .	301
22.3.4	Energy Spectrum in the Tight-Binding Approximation . . . . .	302
22.3.5	Diamagnetic Susceptibility of Bloch Electrons . . . . .	305
22.4	Quantum Oscillations in Magnetic Fields . . . . .	306
22.4.1	Oscillations in a Two-Dimensional Electron Gas . . . . .	307
22.4.2	Energy of a Three-Dimensional Electron Gas in a Magnetic Field . . . . .	310
22.4.3	De Haas–van Alphen Effect . . . . .	314
22.4.4	Role of Spin in Oscillatory Phenomena . . . . .	317
22.4.5	Oscillations in the Magnetization at Finite Temperatures . . . . .	318
22.4.6	Oscillations for General Fermi Surfaces . . . . .	324
22.4.7	Experimental Results . . . . .	325
22.4.8	Further Oscillation Phenomena . . . . .	327
<b>23</b>	<b>Electrons in Thermally Vibrating Lattices . . . . .</b>	<b>329</b>
23.1	Adiabatic Decoupling . . . . .	330
23.2	Hamiltonian of the Electron–Phonon Interaction . . . . .	333
23.2.1	Second-Quantized Form of the Hamiltonian . . . . .	334
23.2.2	Electron–Phonon Matrix Element . . . . .	337
23.2.3	Deformation Potential . . . . .	339
23.2.4	Interaction of Electrons with Optical Phonons . . . . .	341
23.3	Consequences of the Electron–Phonon Interaction . . . . .	343
23.3.1	Finite Lifetime of Electron States . . . . .	344

23.3.2	Polarons	346
23.3.3	Kohn Anomaly and Peierls Instability	350
23.3.4	Jahn–Teller Distortion	353
23.3.5	Effective Electron–Electron Interaction	355
<b>24</b>	<b>Transport Phenomena</b>	<b>357</b>
24.1	General Formulation of Transport Phenomena	358
24.1.1	Currents and Driving Forces	359
24.1.2	Onsager Relations	360
24.2	Boltzmann Equation	361
24.2.1	Nonequilibrium Distribution Function	362
24.2.2	Boltzmann Equation for Electrons	363
24.2.3	Collision Term for Scattering by Lattice Defects	365
24.2.4	Boltzmann Equation for Phonons	366
24.2.5	Coupled Electron–Phonon Systems	368
24.3	Relaxation-Time Approximation	370
24.3.1	Relaxation Time	370
24.3.2	Distribution Function in the Relaxation-Time Approximation	373
24.3.3	DC Conductivity	376
24.3.4	AC and Optical Conductivity	378
24.3.5	General Form of Transport Coefficients	379
24.3.6	Hall Effect	381
24.3.7	Alternative Treatment of Transport in Magnetic Fields	383
24.3.8	Magnetoresistance	384
24.4	Transport Coefficients in Metals and Semiconductors	387
24.4.1	Scattering of Electrons by Impurities	387
24.4.2	Contribution of Electron–Phonon Scattering to the Resistivity	389
24.4.3	Scattering by Magnetic Impurities and the Kondo Effect	394
24.4.4	Electronic Contribution to Thermal Conductivity	399
24.4.5	Phonon Contribution to Thermal Conductivity	400
24.4.6	Transport Coefficients in Semiconductors	402
24.5	Quantum Hall Effect	405
<b>25</b>	<b>Optical Properties of Solids</b>	<b>411</b>
25.1	Interaction of Solids with the Classical Radiation Field	412
25.1.1	Propagation and Absorption of Light in a Dielectric	413
25.1.2	Reflection and Refraction at an Interface	417
25.1.3	Role of Free and Bound Electrons	420
25.1.4	Scattering of Light by Free Electrons	421
25.1.5	Reflectivity of Semiconductors	426
25.1.6	Interaction of Light with Bound Electrons	427
25.1.7	Absorption and Dispersion in Ionic Crystals	430

25.2	Quantum Mechanical Treatment	435
25.2.1	Dielectric Constant of the System of Electrons	435
25.2.2	Electron–Photon Interaction	440
25.2.3	Phonon–Photon Interaction	443
<b>26</b>	<b>Superconductivity</b>	<b>449</b>
26.1	Superconductivity: The Phenomenon	451
26.1.1	Vanishing Resistance, Persistent Current	451
26.1.2	Isotope Effect	452
26.1.3	Meissner–Ochsenfeld Effect	452
26.1.4	Critical Field	455
26.1.5	Thermoelectric Properties	457
26.1.6	Specific Heat	458
26.1.7	Tunneling in SIS and SIN Junctions	460
26.2	Superconducting Materials	461
26.2.1	Superconducting Elements	461
26.2.2	Superconducting Compounds	464
26.2.3	High-Temperature Superconductors	466
26.3	Phenomenological Description of Superconductivity	469
26.3.1	Thermodynamics of Superconductors	470
26.3.2	Electrodynamics of Superconductors	474
26.3.3	Pippard Coherence Length	479
26.3.4	Flux Quantization	481
26.4	Ginzburg–Landau Theory	482
26.4.1	Ginzburg–Landau Equations	483
26.4.2	Gauge Symmetry Breaking	485
26.4.3	Coherence Length and Penetration Depth	486
26.4.4	Flux Quantization	489
26.4.5	Energy of the Normal Metal–Superconductor Interface	491
26.4.6	Vortices	493
26.4.7	Upper and Lower Critical Fields	497
26.5	Josephson Effect	501
26.5.1	Relation Between the Josephson Current and the Phase of the Superconductor	501
26.5.2	DC Josephson Effect	504
26.5.3	AC Josephson Effect	506
26.5.4	Josephson Junctions in a Magnetic Field	509
<b>27</b>	<b>Transport of Carriers in Semiconductor Devices</b>	<b>517</b>
27.1	Interfaces and Junctions	518
27.1.1	Metal–Semiconductor Interface	518
27.1.2	MOS Structures	525
27.1.3	$p$ – $n$ Junctions	526
27.1.4	Heterojunctions	532

27.2	Generation, Motion, and Recombination of Carriers . . . . .	533
27.2.1	Generation and Recombination of Carriers . . . . .	534
27.2.2	Diffusion and Drift of Carriers . . . . .	537
27.2.3	Fundamental Equations of the Physics of Semiconductor Devices . . . . .	540
27.3	Biased Semiconductor Junctions . . . . .	541
27.3.1	Biased Schottky Diodes . . . . .	541
27.3.2	Biased MOS Structures . . . . .	545
27.3.3	Current–Voltage Characteristics of $p$ – $n$ Junctions . . . . .	545
27.3.4	Zener and Avalanche Breakdown in $p$ – $n$ Junctions . . . . .	554
27.3.5	Tunnel Diodes . . . . .	556
27.4	Simple Semiconductor Devices . . . . .	558
27.4.1	Bipolar Transistors . . . . .	558
27.4.2	Field-Effect Transistors . . . . .	560
27.4.3	Semiconductor Lasers and Solar Cells . . . . .	562
27.5	Semiconductor Quantum Devices . . . . .	564
27.5.1	Electron Spectrum of Quantum Wells . . . . .	565
27.5.2	Quantum Wires and Quantum Dots . . . . .	567
27.6	Basics of Spintronics . . . . .	573
<b>G</b>	<b>Quantum Mechanical Perturbation Theory</b> . . . . .	579
G.1	Time-Independent Perturbation Theory . . . . .	579
G.1.1	Nondegenerate Perturbation Theory . . . . .	579
G.1.2	Degenerate Perturbation Theory . . . . .	583
G.2	Time-Dependent Perturbation Theory . . . . .	584
<b>H</b>	<b>Second Quantization</b> . . . . .	589
H.1	Occupation-Number Representation . . . . .	589
H.2	Second-Quantized Form of Operators . . . . .	593
H.2.1	Second-Quantized Form of One-Particle Operators . . . . .	593
H.2.2	Second-Quantized Form of Two-Particle Operators . . . . .	595
H.2.3	Field Operators . . . . .	596
H.2.4	Second-Quantized Form of the Electronic Hamiltonian . . . . .	597
H.2.5	Number-Density and Spin-Density Operators . . . . .	600
<b>I</b>	<b>Canonical Transformation</b> . . . . .	603
I.1	Derivation of an Effective Hamiltonian . . . . .	603
I.1.1	General Formulation of the Problem . . . . .	603
I.1.2	RKKY Interaction . . . . .	605
I.1.3	Derivation of the $s$ – $d$ Interaction . . . . .	611
I.2	Diagonalization of the Hamiltonian . . . . .	612
I.2.1	Bosonic Systems . . . . .	613
I.2.2	Fermionic Systems . . . . .	615

XVIII Contents

<b>Figure Credits</b> .....	617
<b>Name Index</b> .....	621
<b>Subject Index</b> .....	625