Contents

1 Continuum Formulation 1
- 1.1 General Aspects 1
- 1.2 Massless Spin Two Field 1
- 1.3 Wave Equation 2
- 1.4 Feynman Rules 11
- 1.5 One-Loop Divergences 16
- 1.6 Gravity in d Dimensions 21
- 1.7 Higher Derivative Terms 24
- 1.8 Supersymmetry 33
- 1.9 Supergravity 37
- 1.10 String Theory 40
- 1.11 Supersymmetric Strings 48

2 Feynman Path Integral Formulation 55
- 2.1 The Path Integral 55
- 2.2 Sum over Paths 56
- 2.3 Eulidean Rotation 58
- 2.4 Gravitational Functional Measure 59
- 2.5 Conformal Instability 64

3 Gravity in $2+\varepsilon$ Dimensions 67
- 3.1 Dimensional Expansion 67
- 3.2 Perturbatively Non-renormalizable Theories: The Sigma Model 68
- 3.3 Non-linear Sigma Model in the Large-N Limit 79
- 3.4 Self-coupled Fermion Model 83
- 3.5 The Gravitational Case 84
- 3.6 Phases of Gravity in $2+\varepsilon$ Dimensions 92
- 3.7 Running of $\alpha(\mu)$ in Gauge Theories 98

4 Hamiltonian and Wheeler-DeWitt Equation 103
4.1 Classical Initial Value Problem 103
4.2 First Order Formulation 104
4.3 Arnowitt-Deser-Misner (ADM) Formalism 107
4.4 Orthogonal Decomposition in the Linearized Theory 110
4.5 Intrinsic and Extrinsic Curvature, Hamiltonian 111
4.6 Matter Source Terms 112
4.7 Wheeler-DeWitt Equation 114
4.8 Semiclassical Expansion of the Wheeler-DeWitt Equation 115
4.9 Connection with the Feynman Path Integral 117
4.10 Minisuperspace 118
4.11 Solution of Simple Minisuperspace Models 122
4.12 Quantum Hamiltonian for Gauge Theories 127
4.13 Lattice Regularized Hamiltonian for Gauge Theories 129
4.14 Lattice Hamiltonian for Quantum Gravity 135

5 Semiclassical Gravity 141
5.1 Cosmological Wavefunctions 141
5.2 Semiclassical Expansion 146
5.3 Pair Creation in Constant Electric Fields 152
5.4 Black Hole Particle Emission 154
5.5 Method of In and Out Vacua 158
5.6 Complex Periodic Time 163
5.7 Black Hole Evaporation 166
5.8 Quantum Gravity Corrections 167

6 Lattice Regularized Quantum Gravity 169
6.1 The Lattice Theory 169
6.2 General Formulation 170
6.3 Volumes and Angles 171
6.4 Rotations, Parallel Transports and Voronoi Loops 173
6.5 Invariant Lattice Action 179
6.6 Lattice Diffeomorphism Invariance 184
6.7 Lattice Bianchi Identities 186
6.8 Gravitational Wilson Loop 188
6.9 Lattice Regularized Path Integral 190
6.10 An Elementary Example 195
6.11 Lattice Higher Derivative Terms 198
6.12 Scalar Matter Fields 203
6.13 Invariance Properties of the Scalar Action 209
6.14 Lattice Fermions, Tetrads and Spin Rotations 211
6.15 Gauge Fields 213
6.16 Lattice Gravitino 214
6.17 Alternate Discrete Formulations 216
6.18 Lattice Invariance versus Continuum Invariance 220

7 Analytical Lattice Expansion Methods 225
7.1 Motivation 225
7.2 Lattice Weak Field Expansion and Transverse-Traceless Modes 225
7.3 Lattice Diffeomorphism Invariance 234
7.4 Strong Coupling Expansion 242
7.5 Gravitational Wilson Loop 248
7.6 Discrete Gravity in the Large-d Limit 260
7.7 Mean Field Theory 269

8 Numerical Studies 273
8.1 Nonperturbative Gravity 273
8.2 Observables, Phase Structure and Critical Exponents 274
8.3 Invariant Local Gravitational Averages 276
8.4 Invariant Correlations at Fixed Geodesic Distance 278
8.5 Wilson Lines and Static Potential 281
8.6 Scaling in the Vicinity of the Critical Point 284
8.7 Physical and Unphysical Phases 285
8.8 Numerical Determination of the Scaling Exponents 290
8.9 Renormalization Group and Lattice Continuum Limit 295
8.10 Curvature Scales 300
8.11 Gravitational Condensate 302

9 Scale Dependent Gravitational Couplings 305
9.1 Renormalization Group and Scale Dependence of $\boldsymbol{G}$ 305
9.2 Effective Field Equations 305
9.3 Poisson's Equation and Vacuum Polarization Cloud 308
9.4 Static Isotropic Solution 311
9.5 Cosmological Solutions 314
9.6 Quantum Gravity and Mach's Principle 321

References 325

Index 335