

Contents

1	When are High Order Methods Effective?	1
1.1	Preliminaries	1
1.2	Wave Propagation Problems	2
1.3	Parabolic Equations	8
1.4	Schrödinger Type Equations	11
1.5	Summary	12
2	Well-posedness and Stability	13
2.1	Well Posed Problems	13
2.2	Periodic Problems and Fourier Analysis	16
2.2.1	The PDE Problem	17
2.2.2	Difference Approximations	21
2.3	Initial–Boundary Value Problems and the Energy Method	29
2.3.1	The PDE Problem	29
2.3.2	Semidiscrete Approximations	33
2.3.3	Fully Discrete Approximations	38
2.4	Initial–Boundary Value Problems and Normal Mode Analysis for Hyperbolic Systems	41
2.4.1	Semidiscrete Approximations	41
2.4.2	Fully Discrete Approximations	59
2.5	Summary	66
3	Order of Accuracy and the Convergence Rate	69
3.1	Periodic Solutions	69
3.2	Initial–Boundary Value Problems	72
3.3	Summary	79
4	Approximation in Space	81
4.1	High Order Formulas on Standard Grids	81
4.2	High Order Formulas on Staggered Grids	85
4.3	Compact Padé Type Difference Operators	87

4.4	Optimized Difference Operators	91
4.5	Summary	93
5	Approximation in Time	95
5.1	Stability and the Test Equation	95
5.2	Runge–Kutta Methods	97
5.3	Linear Multistep Methods	102
5.4	Deferred Correction	108
5.5	Richardson Extrapolation	111
5.6	Summary	113
6	Coupled Space-Time Approximations	115
6.1	Taylor Expansions and the Lax–Wendroff Principle	115
6.2	Implicit Fourth Order Methods	117
6.3	Summary	124
7	Boundary Treatment	127
7.1	Numerical Boundary Conditions	127
7.2	Summation by Parts (SBP) Difference Operators	130
7.3	SBP Operators and Projection Methods	140
7.4	SBP Operators and Simultaneous Approximation Term (SAT) Methods	147
7.5	Summary	155
8	The Box Scheme	157
8.1	The Original Box Scheme	157
8.2	The Shifted Box Scheme	161
8.3	Two Space Dimensions	165
8.4	Nonuniform Grids	169
8.5	Summary	176
9	Wave Propagation	177
9.1	The Wave Equation	177
9.1.1	One Space Dimension	178
9.1.2	Two Space Dimensions	185
9.2	Discontinuous Coefficients	192
9.2.1	The Original One Step Scheme	193
9.2.2	Modified Coefficients	201
9.2.3	An Example with Discontinuous Solution	206
9.3	Boundary Treatment	209
9.3.1	High Order Boundary Conditions	209
9.3.2	Embedded Boundaries	210
9.4	Summary	216

10 A Problem in Fluid Dynamics 219

 10.1 Large Scale Fluid Problems and Turbulent Flow 219

 10.2 Stokes Equations for Incompressible Flow 220

 10.3 A Fourth Order Method for Stokes Equations 223

 10.4 Navier–Stokes Equations for Incompressible Flow 228

 10.5 A Fourth Order Method for Navier–Stokes Equations 231

 10.6 Summary 242

11 Nonlinear Problems with Shocks 245

 11.1 Difference Methods and Nonlinear Equations 245

 11.2 Conservation Laws 246

 11.3 Shock Fitting 251

 11.4 Artificial Viscosity 252

 11.5 Upwind Methods 257

 11.6 ENO and WENO Schemes 261

 11.7 Summary 265

12 Introduction to Other Numerical Methods 267

 12.1 Finite Element Methods 267

 12.1.1 Galerkin FEM 267

 12.1.2 Petrov–Galerkin FEM 281

 12.2 Discontinuous Galerkin Methods 283

 12.3 Spectral Methods 289

 12.3.1 Fourier Methods 290

 12.3.2 Polynomial Methods 295

 12.4 Finite Volume Methods 300

A Solution of Difference Equations 307

B The Form of SBP Operators 311

 B.1 Diagonal H -norm 311

 B.2 Full H_0 -norm 317

 B.3 A Padé Type Operator 323

References 325

Index 331