

# Table of Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Lower bounding OPT	2
1.1.1	An approximation algorithm for cardinality vertex cover	3
1.1.2	Can the approximation guarantee be improved?	3
1.2	Well-characterized problems and min-max relations	5
1.3	Exercises	7
1.4	Notes	10

---

## Part I. Combinatorial Algorithms

---

<b>2</b>	<b>Set Cover</b>	15
2.1	The greedy algorithm	16
2.2	Layering	17
2.3	Application to shortest superstring	19
2.4	Exercises	22
2.5	Notes	26
<b>3</b>	<b>Steiner Tree and TSP</b>	27
3.1	Metric Steiner tree	27
3.1.1	MST-based algorithm	28
3.2	Metric TSP	30
3.2.1	A simple factor 2 algorithm	31
3.2.2	Improving the factor to $3/2$	32
3.3	Exercises	33
3.4	Notes	37
<b>4</b>	<b>Multiway Cut and <math>k</math>-Cut</b>	38
4.1	The multiway cut problem	38
4.2	The minimum $k$ -cut problem	40
4.3	Exercises	44
4.4	Notes	46

<b>5</b>	<b><i>k</i>-Center</b> .....	47
	5.1 Parametric pruning applied to metric <i>k</i> -center .....	47
	5.2 The weighted version .....	50
	5.3 Exercises .....	52
	5.4 Notes .....	53
<b>6</b>	<b>Feedback Vertex Set</b> .....	54
	6.1 Cyclomatic weighted graphs .....	54
	6.2 Layering applied to feedback vertex set .....	57
	6.3 Exercises .....	60
	6.4 Notes .....	60
<b>7</b>	<b>Shortest Superstring</b> .....	61
	7.1 A factor 4 algorithm .....	61
	7.2 Improving to factor 3 .....	64
	7.2.1 Achieving half the optimal compression .....	66
	7.3 Exercises .....	66
	7.4 Notes .....	67
<b>8</b>	<b>Knapsack</b> .....	68
	8.1 A pseudo-polynomial time algorithm for knapsack .....	69
	8.2 An FPTAS for knapsack .....	69
	8.3 Strong NP-hardness and the existence of FPTAS's .....	71
	8.3.1 Is an FPTAS the most desirable approximation algorithm? .....	72
	8.4 Exercises .....	72
	8.5 Notes .....	73
<b>9</b>	<b>Bin Packing</b> .....	74
	9.1 An asymptotic PTAS .....	74
	9.2 Exercises .....	77
	9.3 Notes .....	78
<b>10</b>	<b>Minimum Makespan Scheduling</b> .....	79
	10.1 Factor 2 algorithm .....	79
	10.2 A PTAS for minimum makespan .....	80
	10.2.1 Bin packing with fixed number of object sizes .....	81
	10.2.2 Reducing makespan to restricted bin packing .....	81
	10.3 Exercises .....	83
	10.4 Notes .....	83
<b>11</b>	<b>Euclidean TSP</b> .....	84
	11.1 The algorithm .....	84
	11.2 Proof of correctness .....	87
	11.3 Exercises .....	89
	11.4 Notes .....	89

---

**Part II. LP-Based Algorithms**


---

<b>12</b>	<b>Introduction to LP-Duality</b> .....	93
12.1	The LP-duality theorem .....	93
12.2	Min-max relations and LP-duality .....	97
12.3	Two fundamental algorithm design techniques .....	100
12.3.1	A comparison of the techniques and the notion of integrality gap .....	101
12.4	Exercises .....	103
12.5	Notes .....	107
<b>13</b>	<b>Set Cover via Dual Fitting</b> .....	108
13.1	Dual-fitting-based analysis for the greedy set cover algorithm .....	108
13.1.1	Can the approximation guarantee be improved? .....	111
13.2	Generalizations of set cover .....	112
13.2.1	Dual fitting applied to constrained set multicover .....	112
13.3	Exercises .....	116
13.4	Notes .....	117
<b>14</b>	<b>Rounding Applied to Set Cover</b> .....	118
14.1	A simple rounding algorithm .....	118
14.2	Randomized rounding .....	119
14.3	Half-integrality of vertex cover .....	121
14.4	Exercises .....	122
14.5	Notes .....	123
<b>15</b>	<b>Set Cover via the Primal-Dual Schema</b> .....	124
15.1	Overview of the schema .....	124
15.2	Primal-dual schema applied to set cover .....	126
15.3	Exercises .....	128
15.4	Notes .....	129
<b>16</b>	<b>Maximum Satisfiability</b> .....	130
16.1	Dealing with large clauses .....	131
16.2	Derandomizing via the method of conditional expectation .....	131
16.3	Dealing with small clauses via LP-rounding .....	133
16.4	A 3/4 factor algorithm .....	135
16.5	Exercises .....	136
16.6	Notes .....	138
<b>17</b>	<b>Scheduling on Unrelated Parallel Machines</b> .....	139
17.1	Parametric pruning in an LP setting .....	139
17.2	Properties of extreme point solutions .....	140
17.3	The algorithm .....	141

17.4	Additional properties of extreme point solutions . . . . .	142
17.5	Exercises . . . . .	143
17.6	Notes . . . . .	144
<b>18</b>	<b>Multicut and Integer Multicommodity Flow in Trees . . . . .</b>	<b>145</b>
18.1	The problems and their LP-relaxations . . . . .	145
18.2	Primal–dual schema based algorithm . . . . .	148
18.3	Exercises . . . . .	151
18.4	Notes . . . . .	153
<b>19</b>	<b>Multiway Cut . . . . .</b>	<b>154</b>
19.1	An interesting LP-relaxation . . . . .	154
19.2	Randomized rounding algorithm . . . . .	156
19.3	Half-integrality of node multiway cut . . . . .	159
19.4	Exercises . . . . .	162
19.5	Notes . . . . .	166
<b>20</b>	<b>Multicut in General Graphs . . . . .</b>	<b>167</b>
20.1	Sum multicommodity flow . . . . .	167
20.2	LP-rounding-based algorithm . . . . .	169
20.2.1	Growing a region: the continuous process . . . . .	170
20.2.2	The discrete process . . . . .	171
20.2.3	Finding successive regions . . . . .	172
20.3	A tight example . . . . .	174
20.4	Some applications of multicut . . . . .	175
20.5	Exercises . . . . .	176
20.6	Notes . . . . .	178
<b>21</b>	<b>Sparsest Cut . . . . .</b>	<b>179</b>
21.1	Demands multicommodity flow . . . . .	179
21.2	Linear programming formulation . . . . .	180
21.3	Metrics, cut packings, and $\ell_1$ -embeddability . . . . .	182
21.3.1	Cut packings for metrics . . . . .	182
21.3.2	$\ell_1$ -embeddability of metrics . . . . .	184
21.4	Low distortion $\ell_1$ -embeddings for metrics . . . . .	185
21.4.1	Ensuring that a single edge is not overshrunk . . . . .	186
21.4.2	Ensuring that no edge is overshrunk . . . . .	189
21.5	LP-rounding-based algorithm . . . . .	190
21.6	Applications . . . . .	191
21.6.1	Edge expansion . . . . .	191
21.6.2	Conductance . . . . .	191
21.6.3	Balanced cut . . . . .	192
21.6.4	Minimum cut linear arrangement . . . . .	193
21.7	Exercises . . . . .	194
21.8	Notes . . . . .	196

<b>22 Steiner Forest</b> .....	197
22.1 LP-relaxation and dual .....	197
22.2 Primal–dual schema with synchronization .....	198
22.3 Analysis .....	203
22.4 Exercises .....	206
22.5 Notes .....	211
<b>23 Steiner Network</b> .....	212
23.1 LP-relaxation and half-integrality .....	212
23.2 The technique of iterated rounding .....	216
23.3 Characterizing extreme point solutions .....	218
23.4 A counting argument .....	220
23.5 Exercises .....	223
23.6 Notes .....	230
<b>24 Facility Location</b> .....	231
24.1 An intuitive understanding of the dual .....	232
24.2 Relaxing primal complementary slackness conditions .....	233
24.3 Primal–dual schema based algorithm .....	234
24.4 Analysis .....	235
24.4.1 Running time .....	237
24.4.2 Tight example .....	237
24.5 Exercises .....	238
24.6 Notes .....	241
<b>25 <math>k</math>-Median</b> .....	242
25.1 LP-relaxation and dual .....	242
25.2 The high-level idea .....	243
25.3 Randomized rounding .....	246
25.3.1 Derandomization .....	247
25.3.2 Running time .....	248
25.3.3 Tight example .....	248
25.3.4 Integrality gap .....	249
25.4 A Lagrangian relaxation technique for approximation algorithms .....	249
25.5 Exercises .....	250
25.6 Notes .....	253
<b>26 Semidefinite Programming</b> .....	255
26.1 Strict quadratic programs and vector programs .....	255
26.2 Properties of positive semidefinite matrices .....	257
26.3 The semidefinite programming problem .....	258
26.4 Randomized rounding algorithm .....	260
26.5 Improving the guarantee for MAX-2SAT .....	263
26.6 Exercises .....	265
26.7 Notes .....	268

---

**Part III. Other Topics**


---

<b>27 Shortest Vector</b> .....	273
27.1 Bases, determinants, and orthogonality defect .....	274
27.2 The algorithms of Euclid and Gauss .....	276
27.3 Lower bounding OPT using Gram–Schmidt orthogonalization .....	278
27.4 Extension to $n$ dimensions .....	280
27.5 The dual lattice and its algorithmic use .....	284
27.6 Exercises .....	288
27.7 Notes .....	292
<b>28 Counting Problems</b> .....	294
28.1 Counting DNF solutions .....	295
28.2 Network reliability .....	297
28.2.1 Upperbounding the number of near-minimum cuts ....	298
28.2.2 Analysis .....	300
28.3 Exercises .....	302
28.4 Notes .....	305
<b>29 Hardness of Approximation</b> .....	306
29.1 Reductions, gaps, and hardness factors .....	306
29.2 The PCP theorem .....	309
29.3 Hardness of MAX-3SAT .....	311
29.4 Hardness of MAX-3SAT with bounded occurrence of variables .....	313
29.5 Hardness of vertex cover and Steiner tree .....	316
29.6 Hardness of clique .....	318
29.7 Hardness of set cover .....	322
29.7.1 The two-prover one-round characterization of <b>NP</b> ....	322
29.7.2 The gadget .....	324
29.7.3 Reducing error probability by parallel repetition .....	325
29.7.4 The reduction .....	326
29.8 Exercises .....	329
29.9 Notes .....	332
<b>30 Open Problems</b> .....	334
30.1 Problems having constant factor algorithms .....	334
30.2 Other optimization problems .....	336
30.3 Counting problems .....	338
30.4 Notes .....	343

**Appendix**

<b>A</b>	<b>An Overview of Complexity Theory for the Algorithm Designer</b> .....	344
	A.1 Certificates and the class <b>NP</b> .....	344
	A.2 Reductions and <b>NP</b> -completeness .....	345
	A.3 <b>NP</b> -optimization problems and approximation algorithms ...	346
	A.3.1 Approximation factor preserving reductions .....	348
	A.4 Randomized complexity classes .....	348
	A.5 Self-reducibility .....	349
	A.6 Notes .....	352
<b>B</b>	<b>Basic Facts from Probability Theory</b> .....	353
	B.1 Expectation and moments .....	353
	B.2 Deviations from the mean .....	354
	B.3 Basic distributions .....	355
	B.4 Notes .....	355
	<b>References</b> .....	357
	<b>Problem Index</b> .....	373
	<b>Subject Index</b> .....	377