Table of Contents

Foreword VII
Preface IX
Introduction 1

Chapter I. Notation, Conventions and Other Preliminaries . 9

§1. Generalities 9
§2. Algebra 12
§3. Topology 14

Chapter II. Spectra and (Co)homology Theories 33

§1. Preliminaries on Spectra 33
§2. The Smash product of Spectra, Duality, Ring and Module Spectra 45
§3. (Co)homology Theories and Their Connection with Spectra 53
§4. Homotopy Properties of Spectra 79
§5. Localization 97
§6. Algebras, Coalgebras and Hopf Algebras 107
§7. Graded Eilenberg–Mac Lane Spectra 120

Chapter III. Phantoms 135

§1. Phantoms and the Inverse Limit Functor 135
§2. Derived Functors of the Inverse Limit Functor 143
§3. Representability Theorems 152
§4. A Spectral Sequence 162
§5. A Sufficient Condition for the Absence of Phantoms 171
§6. Almost Equivalent Spectra (Spaces) 174
§7. Multiplications and Quasi-multiplications 180

Chapter IV. Thom Spectra 185

§1. Fibrations and Their Classifying Spaces 185
§2. Structures on Fibrations 222
§3. A Glance at Locally Trivial Bundles 228
§4. $\mathbb{R}^n$-Bundles and Spherical Fibrations 232
§5. Thom Spaces and Thom Spectra 250
§6. Homotopy Properties of Certain Thom Spectra 268
§7. Manifolds and (Co)bordism 276

Chapter V. Orientability and Orientations 299

§1. Orientations of Bundles and Fibrations 300
§2. Orientations of Manifolds 316
§3. Orientability and Integrality 323
§4. Obstructions to Orientability 327
§5. Realizability of the Obstructions to Orientability 335

Chapter VI. K and $K\mathcal{O}$-Orientability 339

§1. Some Secondary Operations on Thom Classes 339
§2. Some Calculations with Classifying Spaces 351
§3. k-Orientability 359
§4. $k\mathcal{O}$-Orientability 374
§5. A Few Geometric Observations 379

Chapter VII. Complex (Co)bordism 383

§1. Homotopy and Homology Properties of the Spectrum $M\mathcal{U}$. 383
§2. $\mathbb{C}$-oriented Spectra 393
§3. Operations on $M\mathcal{U}$. Idempotents. The Brown–Peterson Spectrum BP 402
§4. Invariant Prime Ideals. The Filtration Theorem 419
§5. Formal Groups 428
§6. Formal Groups Input 433
§7. The Steenrod–tom Dieck Operations 446

Chapter VIII. (Co)bordism with Singularities 457

§1. Definitions and Basic Properties 457
§2. Multiplicative Structures 466
§3. Obstructions and the Steenrod–tom Dieck Operations 474
§4. A Universality Theorem for $M\mathcal{U}$ with Singularities 481
§5. Realization of Homology Classes by PL Manifolds with Singularities 487

Chapter IX. Complex (Co)bordism with Singularities 495

§1. Brown–Peterson (Co)homology with Singularities 495
§2. The Spectra $P(n)$ 498
§3. Homological Properties of the Spectra $P(n)$ 505
§4. The Exactness Theorem 514
§5. Commutative Ring Spectra of Characteristic 2 521
§6. The Spectra $BP\langle n\rangle$ and Homological Dimension 528
§7. Morava K-Theories 538

References 553

List of Notations 573

Subject Index 579