Contents

1 Basic Theories and Principles of Nonlinear Beam Deformations 1
1.1 Introduction 1
1.2 Brief Historical Developments Regarding the Static and the Dynamic Analysis of Flexible members 1
1.3 The Euler–Bernoulli Law of Linear and Nonlinear Deformations for Structural Members 8
1.4 Integration of the Euler–Bernoulli Nonlinear Differential Equation 10
1.5 Simpson's One-Third Rule 14
1.6 The Elastica Theory 16
1.7 Moment and Stiffness Dependence on the Geometry of the Deformation of Flexible Members 22
1.8 General Theory of the Equivalent Systems for Linear and Nonlinear Deformations 29
1.8.1 Nonlinear Theory of the Equivalent Systems: Derivation of Pseudolinear Equivalent Systems 30
1.8.2 Nonlinear Theory of the Equivalent Systems: Derivation of Simplified Nonlinear Equivalent Systems 40
1.8.3 Linear Theory of the Equivalent Systems 44

2 Solution Methodologies for Uniform Flexible Beams 63
2.1 Introduction 63
2.2 Pseudolinear Analysis for Uniform Flexible Cantilever Beams Loaded with Uniformly Distributed Loading Throughout their Length 64
2.3 Pseudolinear Analysis for Uniform Simply Supported Beams Loaded with a Uniformly Distributed Loading Throughout their Length 71
2.4 Flexible Uniform Simply Supported Beam Loaded with a Vertical Concentrated Load 76

2.5 Uniform Statically indeterminate Single Span Flexible Beam Loaded with a Uniformly Distributed Load w_o on its Entire Span 82
2.6 Uniform Statically Indeterminate Single Span Flexible Beam Subjected to a Vertical Concentrated Load 86
2.7 Flexible Uniform Cantilever Beam Under Combined Loading Conditions 90
2.8 Flexible Uniform Cantilever Beam Under Complex Loading Conditions 96
2.8.1 Application of Equivalent Pseudolinear Systems 96
2.8.2 Deriving Simpler Nonlinear Equivalent Systems 101

3 Solution Methodologies for Variable Stiffness Flexible Beams 105
3.1 Introduction 105
3.2 Flexible Tapered Cantilever Beam with a Concentrated Vertical Load at its Free End 105
3.3 Doubly Tapered Flexible Cantilever Beam Subjected to a Uniformly Distributed Loading 113
3.4 Solution of the Problem in the Preceding Section by Using a Simplified Nonlinear Equivalent System 119
3.5 Flexible Tapered Simply Supported Beam with Uniform Load 121
3.6 Flexible Tapered Simply Supported Beam Carrying a Trapezoidal Load 125
3.7 Using an Alternate Approach to Derive a Simpler Equivalent Nonlinear System of Constant Stiffness 128
3.7.1 Application to Cantilever Flexible Beam Problems 129
3.7.2 Application to Flexible Simply Supported Beam Problems 133

4 Inelastic Analysis of Structural Components 143
4.1 Introduction 143
4.2 Theoretical Aspects of Inelastic Analysis 144
4.2.1 The Theory and Concept of the Reduced Modulus E_r 144
4.2.2 Application of the Method of the Equivalent Systems for Inelastic Analysis 155
4.3 Inelastic Analysis of Simply Supported Beams 165
4.4 Ultimate Design Loads Using Inelastic Analysis 172

5 Vibration Analysis of Flexible Structural Components 185
5.1 Introduction 185
5.2 Nonlinear Differential Equations of Motion 186
5.2.1 The general Nonlinear Differential Equation of Motion 186
5.2.2 Small Amplitude Vibrations of Flexible Members 189

5.3 Application of the Theory and Method 193
5.3.1 Free Vibration of Uniform Flexible Cantilever Beams 193
5.3.2 Free Vibration of Flexible Simply supported Beams 204
5.4 The Effect of Mass Position Change During the Vibration of Flexible Members 211
5.5 Galerkin's Finite Element Method (GFEM) 213
5.6 Vibration of Tapered Flexible Simply Supported Beams Using Galerkin's FEM 221
5.7 Concluding Remarks 224

6 Suspension Bridges, Failures, Plates, and Other Types of Nonlinear Structural Problems 229
6.1 Introduction 229
6.2 Brief Discussion on Fundamental Aspects of Suspension Bridges 229
6.3 The Collapse of the Tacoma Narrows Suspension Bridge 232
6.4 Other Failures and What We Learn from Them 235
6.5 Eccentrically Loaded Columns 237
6.6 Inelastic Analysis of Members with Axial Restraints Using Equivalent Systems 240
6.7 The Longest Cable-Stayed Suspension Bridge in the World . . . 253
6.8 Inelastic Analysis of Thin Rectangular Plates 259
6.9 Inelastic Earthquake Response of Multistory Buildings 269
6.9.1 Resistant R of a Structure 270
6.9.2 Multistory Buildings Subjected to Strong Earthquakes 276
6.10 Elastic and Inelastic Analysis of Thick-Walled Cylinders Subjected to Uniform External and Internal Pressures 285
6.10.1 Elastic Analysis of Thick Cylinders 285
6.10.2 Inelastic Analysis of Thick Cylinders 290
6.11 Inelastic Analysis of Members of Non-Rectangular Cross Sections 294
6.12 Torsion Beyond the Elastic Limit of the Material 297
6.13 Vibration Analysis of Inelastic Structural Members 299
6.14 Inelastic Analysis of Flexible Members 307

Appendix A Acceleration Impulse Extrapolation Method (AIEM) 323

Appendix B Computer Program Using the AIEM for the Elastoplastic Analysis in Example 6.5 327

References 329