

Contents

1	Compact Objects in Astrophysics	1
1.1	Why is Newtonian Gravity Obsolete?	1
1.2	Einstein was Skeptical about the Existence of Black Holes	3
1.3	Subrahmanyan Chandrasekhar and Compact Objects	4
1.4	Classes of Compact Objects	5
1.4.1	White Dwarfs and Neutron Stars	8
1.4.2	Compact X-Ray Sources	9
1.4.3	Radio Pulsars	11
1.5	Supermassive Black Holes in Galactic Centers	16
1.6	Gamma-Ray Bursters	19
	Problems	25
2	Gravity of Compact Objects	27
2.1	Geometric Concepts and General Relativity	27
2.2	The Basic Principles of General Relativity	29
2.2.1	Einstein's Equivalence Principle and Metricity	29
2.2.2	Metric Theories of Gravity	33
2.3	Basic Calculus on Manifolds	37
2.3.1	Tensors and Forms on Manifolds	37
2.3.2	The Metric Field and Pseudo-Riemannian Manifolds	42
2.3.3	The Calculus of Forms on Lorentzian Manifolds	44
2.4	Affine Connection and Covariant Derivative	47
2.4.1	Affine Connection	47
2.4.2	Covariant Derivative of Vector Fields	47
2.4.3	Covariant Derivative for Tensor Fields	48
2.4.4	Parallel Transport and Metric Connection	50
2.4.5	Metric Connection	52
2.4.6	Divergence of Vector Fields	55
2.5	Curvature of Pseudo-Riemannian Manifolds	56
2.5.1	Mathematical Definition of Torsion and Curvature	57
2.5.2	Bianchi Identities for Metric Connection	58
2.5.3	Ricci, Weyl and Einstein Tensor	60
2.5.4	Cartan's Structure Equations	61
2.6	Gravity is a Lorentzian Connection on Spacetime	65
2.6.1	The Four Key Principles of General Relativity	65

2.6.2	The Hilbert Action and Einstein’s Field Equations	68
2.6.3	On the Cosmological Constant	69
2.6.4	Limits of General Relativity	71
2.7	Gravitational Waves	73
2.7.1	The Geodesic Deviation – Relativistic Tidal Forces	73
2.7.2	Gravity Wave Experiments	74
2.7.3	The Nature of Gravitational Waves	76
2.7.4	Degrees of Freedom	79
2.7.5	Gravitational Wave Solutions	83
2.7.6	The Quadrupole Formula	87
2.8	3+1 Split of Einstein’s Equations	91
2.8.1	Induced Spatial Metric and Extrinsic Curvature	92
2.8.2	Hypersurface Embedding	93
2.8.3	Split of Affine Connection and Curvature	95
2.8.4	Split of Einstein’s Equations	98
2.8.5	Black Hole Simulations and Gravitational Waves	100
	Problems	101
3	Matter Models for Compact Objects	105
3.1	General Relativistic Hydrodynamics	105
3.1.1	Relativistic Plasma Equations	106
3.1.2	On Numerics of Hydrodynamics	110
3.2	The Boltzmann Equation in GR	113
3.2.1	The Geodesics Spray on the Cotangent Bundle	113
3.2.2	Particle Number Current and Energy–Momentum Tensor	116
3.2.3	The Relativistic Boltzmann Equation	117
3.2.4	Liouville Operator in 3+1 Split	118
3.2.5	Transformation into the Local Rest Frame	119
	Problems	120
4	Relativistic Stellar Structure	123
4.1	Spacetime of Relativistic Stars	123
4.2	Derivation of the TOV Equations	125
4.2.1	The Curvature of Static Spacetimes	125
4.2.2	Matter in the Interior	127
4.2.3	The Exterior Schwarzschild Solution	130
4.2.4	Stable Branches for Degenerate Stars	131
4.2.5	Metric for Relativistic Stars	131
4.3	A Variational Principle for the Stellar Structure	132
	Problems	134

5	White Dwarfs	137
5.1	Observations of Isolated White Dwarfs	138
5.1.1	Sirius B	138
5.1.2	Field White Dwarfs and Classification	139
5.1.3	White Dwarfs in Globular Clusters	143
5.1.4	Magnetic White Dwarfs	143
5.1.5	Ultracool White Dwarfs as Cosmochronometers	145
5.2	What is Inside a White Dwarf?	151
5.3	Equation of State below the Neutron Drip Density	153
5.4	Structure of White Dwarfs and the Chandrasekhar Mass	159
5.4.1	Polytropic Approximation	160
5.4.2	Beyond the Chandrasekhar Treatment	162
5.4.3	Comparison with Observations	162
5.5	The Relativistic Instability of White Dwarf Stars	167
5.5.1	Necessary Condition for Stability	168
5.5.2	The Total Energy in the Post-Newtonian Limit	169
5.5.3	GR White Dwarf Instability	171
5.6	Cooling White Dwarfs	174
5.6.1	Structure of the Surface Layers	175
5.6.2	Cooling Curves and Crystallization	177
5.6.3	Testing WD Crystallization Theory	179
5.7	White Dwarfs in Binary Systems	180
	Problems	185
6	Neutron Stars	187
6.1	The Structure of a Neutron Star	188
6.2	Equations of State beyond Neutron Drip	189
6.2.1	From Neutron Drip to Saturation	190
6.2.2	Nuclear EoS for Dense Neutron Matter	199
6.2.3	Relativistic Mean Field Theory above Saturation	206
6.2.4	Analytical Fits to EoS	216
6.3	Neutron Star Models	219
6.3.1	Hadronic Models	219
6.3.2	Quark Matter Cores	224
6.3.3	Grand Canonical Potential for Quark Matter	231
6.3.4	Strange Quark Stars	241
6.3.5	The Structure of Massive Neutron Stars	242
6.4	Neutron Stars in Close Binary Systems	244
6.4.1	Post-Newtonian Potentials for Many-Body Systems	244
6.4.2	Periastron Shift in Two-Body Systems	248
6.4.3	The Shapiro Time Delay in a Binary System	250
6.4.4	Decay of Binary Orbits due to Gravitational Radiation	251
6.5	Masses of Neutron Stars from Radio Pulsar Timing	255
6.5.1	What is Pulsar Timing?	255
6.5.2	The Timing Formula	259

6.5.3	Timing of the Binary System PSR B1913+16	263
6.5.4	Masses of Companion Stars	264
6.5.5	The Double Pulsar System PSR 0737-3039A+B	265
6.6	Neutron Stars in our Galaxy	269
6.6.1	100 Million Neutron Stars in the Galaxy	269
6.6.2	Thermal Emission from Isolated Neutron Stars	272
6.6.3	Rotation-Powered Pulsars	284
6.6.4	Accretion-Powered Neutron Stars and the Mass–Radius Relation	294
	Problems	303
7	Rapidly Rotating Neutron Stars	307
7.1	Spacetime of Stationary and Axisymmetric Rotating Bodies	308
7.1.1	Physical Interpretation of the Metric	309
7.1.2	Geodetic and Lense–Thirring Precession	312
7.1.3	On General 3+1 Split of Spacetime	315
7.2	Einstein’s Field Equations for Rotating Objects	317
7.2.1	Ricci Tensors of Time-Slices	318
7.2.2	Extrinsic Curvature and 4D Ricci Tensors	319
7.2.3	3+1 Split of Einstein’s Equations	320
7.3	Stellar Structure Equations in Isotropic Gauge	321
7.3.1	The Isotropic Gauge	321
7.3.2	Structure Equations for Rotating Stars	322
7.3.3	Mechanical Equilibrium and Effective Potential	324
7.3.4	Stellar Parameters	326
7.4	The Slow-Rotation Approximation	332
7.5	Numerical Integration of the Stellar Structure Equations	335
7.5.1	Comparison of Numerical Codes	337
7.5.2	Properties of Rotating Equilibrium Stellar Structures	338
7.6	Towards Analytical Vacuum Solutions for Rotating Neutron Stars	342
7.6.1	Weyl–Papapetrou Form	342
7.6.2	Ernst Equations	343
7.6.3	Manko’s Solution	345
7.7	On Oscillation and Formation of Rotating Neutron Stars	350
	Problems	353
8	Black Holes	355
8.1	The Schwarzschild Black Hole	355
8.1.1	Tortoise Coordinates and Null Cones	356
8.1.2	Roads towards Black Hole Formation	358
8.1.3	The Kruskal Extension	359
8.1.4	Penrose Diagram – the Conformal Structure of Infinity	363
8.2	Geodetic Motions in Schwarzschild Spacetime	369
8.2.1	A Lagrangian	369
8.2.2	The Effective Potential for Equatorial Motion	371

8.2.3	Orbital Equation and Bound Orbits in Schwarzschild Spacetime	373
8.3	The Kerr Black Hole	378
8.3.1	Kerr Black Hole in Boyer–Lindquist Coordinates	379
8.3.2	A Short Derivation of the Kerr Solution	379
8.3.3	The Weyl–Papapetrou Form of the Kerr Metric	384
8.3.4	Uniqueness of the Kerr Solution	385
8.3.5	Global Properties of the Kerr Metric	386
8.3.6	On the Conformal Structure of the Kerr Solution	393
8.3.7	Ernst’s Equations for the Kerr Geometry	394
8.3.8	The Kerr–Schild Metric and Two-Black-Hole States	395
8.4	Rotational Energy and the Four Laws of Black Hole Evolution	399
8.4.1	Surface Gravity and Angular Velocity of the Horizon	400
8.4.2	First Law of Black Hole Dynamics	402
8.4.3	Rotational Energy of Astrophysical Black Holes	405
8.4.4	On the Second and Third Laws of Black Hole Dynamics ...	406
8.5	Time Evolution of Black Holes	408
8.5.1	Quasistationary Evolution of Accreting Black Holes	408
8.5.2	Merging of Black Holes	411
8.6	Geodesics in the Kerr Geometry	412
8.6.1	Direct Integration of Geodesics Equations	414
8.6.2	Geodesics in the Equatorial Plane	416
8.6.3	Geodesics Including Lateral Motion	424
8.6.4	Null Geodesics and Ray-Tracing in Kerr Geometry	431
8.7	Dark Energy Stars	442
8.7.1	Why Dark energy Stars?	442
8.7.2	Structure of Gravastars	443
8.7.3	The Necessity of an Anisotropic Crust	445
	Problems	446
9	Astrophysical Black Holes	449
9.1	Classes of Astrophysical Black Holes	450
9.2	Measuring Black Hole Masses	451
9.2.1	BHs in X-Ray Binaries	451
9.2.2	Intermediate-Mass Black Holes	456
9.2.3	Supermassive Black Holes in Nearby Galaxies	456
9.2.4	Black Holes in Quasars	468
9.3	Estimating Black Hole Spin	470
9.3.1	Black Hole Spin and Radio Galaxies	471
9.3.2	Spectral Fitting of Accretion Disks	471
9.3.3	Relativistic Iron Lines	472
9.3.4	Quasiperiodic Oscillations	472
9.4	Black Holes and Galaxy Formation	472
9.5	Black Hole Magnetospheres	473
9.5.1	The 3+1 Formalism for Maxwell’s Equations	474

9.5.2	Plasma Equations in the 3+1 Split	478
9.5.3	Time Evolution of Magnetic and Current Flux in Turbulent Disks	480
9.5.4	Stationary Magnetospheres on Kerr Black Holes	486
9.5.5	Relaxation of Black Hole Magnetospheres and the Blandford–Znajek Process	499
9.6	Magnetic Spin-Down of Rotating Black Holes	509
	Problems	511
10	Physics of Accretion Flows around Compact Objects	513
10.1	Angular Momentum Transport	514
10.2	Magnetohydrodynamics for Accretion Disks	517
10.2.1	Equations of Magnetohydrodynamics	517
10.2.2	Time and Space Discretization	523
10.2.3	MRI Driven Turbulence in Disks	525
10.2.4	Two-Temperature Plasmas and Radiation Pressure in Accretion Disks	533
10.3	States of Turbulent Accretion Disks	537
10.3.1	Turbulent Angular Momentum Transport in Accretion Disks	538
10.3.2	Truncated Accretion and Standard Disk Models in 1D	540
10.3.3	Standard Thin Disk Solutions (SSD)	545
10.3.4	Advection-Dominated Flows (ADAF)	551
10.3.5	Super-Eddington Accretion	552
10.3.6	Unified Models of Disk Accretion	553
10.3.7	Fundamental Time-Scales for Accreting Black Holes	555
10.4	Relativistic MHD – Turbulent Accretion onto Black Holes	558
10.4.1	From SRMHD to GRMHD	558
10.4.2	The Equations for GRMHD	559
10.4.3	Nonradiative Accretion onto Rotating Black Holes	563
10.5	Jets and the Ergosphere	565
10.5.1	Jets as Outflows from the Ergospheric Region	566
10.5.2	From the Ergosphere to the Cluster Gas	572
	Problems	575
11	Epilogue and Future Prospects	579
	Astrophysical Constants and Symbols	587
	SLy4 Equation of State for Neutron Star Matter	591
3+1 Split of Spacetime Curvature		595
C.1	Gauss Decomposition	595
C.2	Codazzi–Mainardi Equations	596

3+1 Split of Rotating Neutron Star Geometry	599
D.1 The 3+1 Split of the Connection	599
D.2 The Curvature of Time Slices	601
Equations of GRMHD	605
E.1 Electromagnetic Fields	605
E.2 Conservative Formulation of GRMHD	607
E.3 Numerical Schemes	609
Solutions	613
Glossary	641
References	657
Index	675