

# Contents

## **An Introduction to Varying Fundamental Constants**

<i>Savely G. Karshenboim, Ekkehard Peik</i> .....	1
1 Introduction .....	1
2 Fundamental Constants .....	3
3 Variability of Fundamental Constants .....	5
3.1 ‘Fundamentality’ of the Fundamental Constants .....	6
3.2 ‘Constancy’ of the Fundamental Constants .....	7
3.3 Intercorrelations Between the Fundamental Constants .....	7
3.4 Variability of Fundamental Constants and Equivalence Principle ..	8
4 Astrophysical and Geophysical Search for a Variability of Constants ..	9
5 New Frequency Standards and Constraints on Variation of Fundamental Constants .....	10
6 Summary: Results and Open Questions .....	15

---

## **Part I Astrophysics**

---

### **Time and the Universe**

<i>Gerhard Börner</i> .....	21
1 Introduction .....	21
2 The Cosmological Models .....	22
3 The History of the Universe .....	25
4 The Cosmic Microwave Background .....	26
5 The Inflationary Model .....	30
6 Variation of the Fine Structure Constant .....	31
7 Conclusions .....	31

### **Millisecond Pulsars as Tools of Fundamental Physics**

<i>Michael Kramer</i> .....	33
1 Introduction .....	33
2 Pulsars .....	34
2.1 Pulsars as Neutron Stars .....	35
2.2 Pulsars as Radio Sources .....	35
3 A Pulsar’s Life .....	36
3.1 Normal Pulsars .....	37
3.2 Millisecond Pulsars .....	37

X	Contents	
4	Pulsars as Clocks	38
4.1	Time Transfer	38
4.2	Pulsar Timing	42
5	Applications of Pulsars	44
5.1	PPN Parameters	44
5.2	Tests Using Double Neutron Stars	48
5.3	Tests Using Profile Structure Data	50
5.4	Recent Discoveries	51
6	The Double-Pulsar	51
7	Conclusions and Outlook	53

---

## Part II Fundamental Constants

---

### Fundamental Units: Physics and Metrology

	<i>Lev B. Okun</i>	57
1	Introduction	57
2	Fundamental Parameters and Units	58
3	Planck Units	58
4	$c$ , $h$ , $G$ – Units	59
5	Planck Units Are Impractical	59
6	Units of Stoney	63
7	Atomic Clocks and $c$	66
8	Towards a Kilogram Based on $h$	66
9	Kilogram as Frequency $\nu_K$	68
10	Electromagnetism and Relativity	68
11	Concluding Remarks	71

### Constants, Units and Standards

	<i>Jeff Flowers, Brian Petley</i>	75
1	Introduction	75
1.1	Early Measurements	76
1.2	The Fundamental Physical Constants	76
2	Units and Standards	78
2.1	Use of the Fundamental Constants to Form Systems of Units	79
2.2	Are the Constants Really Constant?	79
2.3	The CODATA Evaluations	79
2.4	Changing Accuracy	80
3	Accuracy of Realization of the SI Units	82
4	Practical Realizations of the SI Units and the Involvement of Fundamental Constants	82
4.1	The Josephson Effect Voltage Standard	83
4.2	The Quantised Hall Resistance	83
4.3	The Calculable Capacitor	83
4.4	The Moving Coil Watt Realization of Kibble	84
4.5	The Kilogram	85

4.6	The Anomalous $g$ -Factor of the Electron .....	85
4.7	The Rydberg Constant .....	86
4.8	The Newtonian Constant of Gravitation .....	87
5	Underpinning of the SI by the Fundamental Physical Constants .....	87
5.1	The Importance of the Fine Structure Constant in Metrology....	87
6	Conclusion .....	90
6.1	Future .....	91
7	Summary.....	92

### Part III    Grand Unification and Quantum Gravity

#### Time Varying Fundamental Constants, Extra Dimensions and the Renormalization Group

<i>William J. Marciano</i> .....		97
1	Dirac Revisited – The Hierarchy Problem .....	97
2	Fundamental Constants from a Modern Perspective.....	99
3	Extra Dimensions .....	101
4	Renormalization Group Connections .....	102
5	Examples .....	103
6	Discussions .....	104

#### Fundamental Constants and Their Possible Time Dependence

<i>Harald Fritzsch</i> .....		107
1	Introduction .....	107
2	Variation of Fundamental Constants and Grand Unification.....	110

#### Quantum Gravity and Fundamental Constants

<i>Claus Kiefer</i> .....		115
1	Introduction .....	115
2	Quantum General Relativity .....	118
3	Superstring Theory ('M-theory') .....	121
4	Kaluza–Klein Theories .....	124
5	Conclusion .....	126

### Part IV    Astrophysical and Geochemical Search

#### Constraining Variations in the Fine-Structure Constant, Quark Masses and the Strong Interaction

	<i>Michael T. Murphy, Victor V. Flambaum, John K. Webb</i> <i>Vladimír V. Dzuba, Jason X. Prochaska, Arthur M. Wolfe</i> .....	131
1	Introduction .....	131
2	Varying $\alpha$ from Quasar Absorption Lines .....	132
2.1	Quasar Absorption Lines .....	132

2.2	The Many-Multiplet (MM) Method	132
2.3	Spectral Analysis and Updated Results	135
2.4	Recent Criticisms of the MM Method	139
2.5	Isotopic Abundance Variations	142
3	Varying $\alpha$ and $m_q/\Lambda_{\text{QCD}}$ from Atomic Clocks	143
3.1	Introduction	143
3.2	Nuclear Magnetic Moments, $\alpha$ and $m_q/\Lambda_{\text{QCD}}$	144
3.3	Results	146
4	Conclusions	148

**Astrophysical Constraints on Hypothetical Variability of Fundamental Constants**

<i>Sergei A. Levshakov</i>		151
1	Introduction	151
2	Methods to Constrain $\Delta\alpha/\alpha$	
	from QSO Absorption Spectra	153
2.1	The Alkali-Doublet (AD) Method	153
2.2	The Many-Multiplet (MM) Method	154
2.3	The Regression MM Method	158
3	Constraints on the Proton-to-Electron Mass Ratio	160
4	Conclusions and Future Prospects	163

**Oklo Constraint on the Time-Variability of the Fine-Structure Constant**

<i>Yasunori Fujii</i>		167
1	What Is the Oklo Phenomenon?	167
2	How Did Shlyakhter Probe $\Delta\alpha$ ?	168
3	How Good Is It?	170
4	How Can It Be Consistent with the QSO Result?	174
A	Bound on $\Delta\alpha/\alpha$ from the Coulomb-Only Estimate	181
B	Distant Migration of the Higher Resonances	182
C	Another 3-Parameter Fit with an Offset	184

---

**Part V Precision Frequency Measurements with Neutral Atoms**

---

**Cold Atom Clocks, Precision Oscillators and Fundamental Tests**

<i>S. Bize, P. Wolf, M. Abgrall, L. Cacciapuoti, A. Clairon, J. Grünert, Ph. Laurent, P. Lemonde, A.N. Luiten, I. Maksimovic, C. Mandache, H. Marion, F. Pereira Dos Santos, P. Rosenbusch, C. Salomon, G. Santarelli, Y. Sortais, M.E. Tobar, C. Vian, S. Zhang</i>		189
1	Introduction	189
2	Test of Local Position Invariance.	
	Stability of Fundamental Constants	190
2.1	Theory	190
2.2	Experiments with $^{87}\text{Rb}$ and $^{133}\text{Cs}$ Fountain Clocks	193

3 Tests of Local Lorentz Invariance ..... 197  
 3.1 Theory ..... 198  
 3.2 Experimental Results ..... 201  
 4 Conclusion and Outlook ..... 204

**Precision Spectroscopy of Atomic Hydrogen and Variations of Fundamental Constants**

*M. Fischer, N. Kolachevsky, M. Zimmermann, R. Holzwarth, Th. Udem, T.W. Hänsch, M. Abgrall, J. Grünert, I. Maksimovic, S. Bize, H. Marion, F. Pereira Dos Santos, P. Lemonde, G. Santarelli, P. Laurent, A. Clairon, C. Salomon* ..... 209  
 1 Introduction ..... 209  
 2 Hydrogen Spectrometer ..... 212  
 3 Frequency Measurement ..... 215  
 4 Determination of Drift Rates ..... 221  
 5 Conclusion ..... 225

**An Optical Frequency Standard with Cold and Ultra-cold Calcium Atoms**

*Fritz Riehle, Carsten Degenhardt, Christian Lisdat, Guido Wilpers, Harald Schnatz, Tomas Binnewies, Hardo Stoehr, Uwe Sterr* ..... 229  
 1 Introduction ..... 229  
 2 Methods and Experimental Realization ..... 230  
 2.1 Properties of the Calcium Standard ..... 230  
 2.2 Production of Cold Ca Atoms ( $T \approx 3$  mK) ..... 231  
 2.3 Production of Ultra-cold Atoms ..... 232  
 2.4 Interrogation of the Clock Transition ..... 232  
 3 Uncertainty of the Optical Ca Frequency Standard ..... 235  
 3.1 Residual First-order Doppler Shifts ..... 235  
 3.2 Other Phase Shifts ..... 236  
 3.3 Frequency Shifts Due to External Fields ..... 236  
 3.4 Influence of Cold and Ultra-cold Atomic Collisions ..... 240  
 3.5 Uncertainty Budget ..... 240  
 4 Frequency Measurements ..... 241  
 5 Prospects of the Ca Optical Frequency Standard ..... 242

---

**Part VI Frequency Standards with a Single Trapped Ion**

---

**Trapped Ion Optical Frequency Standards for Laboratory Tests of Alpha-Variability**

*Christian Tamm, Tobias Schneider, Ekkehard Peik* ..... 247  
 1 Introduction ..... 247  
 2 The Single Ion as a Reference in an Optical Clock ..... 248  
 3 Spectroscopy of the 435.5 nm Clock Transition of  $^{171}\text{Yb}^+$  ..... 250

4	Absolute Transition Frequency and Frequency Comparison Between Two Ions . . . . .	251
5	Search for Temporal Variation of the Fine-Structure Constant . . . . .	254
6	Nuclear Optical Frequency Standard with Th-229 . . . . .	257

**An Optical Frequency Standard Based on the Indium Ion**

*Petrissa Eckle, Mario Eichenseer, Alexander Y. Nevsky,  
Christian Schwedes, Joachim von Zanthier, Herbert Walther . . . . .*

		263
1	Introduction . . . . .	263
2	Cooling of the Indium Ion . . . . .	265
3	New Cooling Laser System . . . . .	265
4	High-Resolution Spectroscopy . . . . .	268
5	Absolute Frequency Measurements . . . . .	269

**Part VII High-Resolution Molecular Spectroscopy**

**Applications of Femtosecond Laser Comb  
to Nonlinear Molecular Spectroscopy**

*Jun Ye, R. Jason Jones, Lisheng Chen, Kevin W. Holman,  
David J. Jones . . . . .*

		275
1	Introduction to Femtosecond Optical Frequency Comb . . . . .	275
2	Molecular Spectroscopy Aided by Femtosecond Optical Frequency Comb . . . . .	281
3	$I_2$ Hyperfine Interactions, Optical Frequency Standards and Clocks . . .	283
4	Extend Phase-Coherent fs Combs to the Mid-IR Spectral Region . . . . .	288
5	Femtosecond Lasers and External Optical Cavities . . . . .	290

**Ultracold Trapped Molecules:**

**Novel Systems for Tests of the Time-Independence  
of the Electron-to-Proton Mass Ratio**

*U. Fröhlich, B. Roth, P. Antonini, C. Lämmerzahl, A. Wicht,  
S. Schiller . . . . .*

		297
1	Introduction . . . . .	297
2	Molecular Tests of Constancy of Electron-to-Nucleon Mass Ratios . . . .	299
3	Sympathetic Cooling of Molecular Ions and Spectroscopy . . . . .	301
4	Quantum Jump Spectroscopy . . . . .	304
5	Conclusion . . . . .	306

---

**Part VIII Space Missions and General Relativity**

---

**35 Years of Testing Relativistic Gravity:  
Where Do We Go from Here?**

*Slava G. Turyshev, James G. Williams, Kenneth Nordtvedt, Jr.,  
Michael Shao, Thomas W. Murphy, Jr. . . . .* 311

1 Introduction . . . . . 311

2 Scientific Motivation . . . . . 313

    2.1 PPN Parameters and Their Current Limits . . . . . 313

    2.2 Motivations for Precision Gravity Experiments . . . . . 314

3 Lunar Laser Ranging: A Unique Laboratory in Space . . . . . 318

    3.1 LLR History and Scientific Background . . . . . 318

    3.2 Equivalence Principle Tests . . . . . 319

    3.3 LLR Tests of the Equivalence Principle . . . . . 321

    3.4 LLR Tests of Other Gravitational Physics Parameters . . . . . 322

    3.5 APOLLO Contribution to the Tests of Gravity . . . . . 323

4 New Test of Relativity: The LATOR Mission . . . . . 324

    4.1 Overview of LATOR . . . . . 324

    4.2 The Expected Results from LATOR . . . . . 326

5 Conclusions . . . . . 328

**Search for New Physics with Atomic Clocks**

*Lute Maleki and John Prestage . . . . .* 331

1 Introduction . . . . . 331

2 The Instrument . . . . . 334

    2.1 Temperature Induced Frequency Shifts . . . . . 338

    2.2 Mission Design . . . . . 339

3 Conclusion . . . . . 340

Index . . . . . 342

**Index . . . . .** 343