
Contents

1	Introduction	1
1.1	A Database Example	1
1.2	An Example from Complexity Theory	4
1.3	An Example from Formal Language Theory	6
1.4	An Overview of the Book	8
1.5	Exercises	10
2	Preliminaries	13
2.1	Background from Mathematical Logic	13
2.2	Background from Automata and Computability Theory	17
2.3	Background from Complexity Theory	19
2.4	Bibliographic Notes	21
3	Ehrenfeucht-Fraïssé Games	23
3.1	First Inexpressibility Proofs	23
3.2	Definition and Examples of Ehrenfeucht-Fraïssé Games	26
3.3	Games and the Expressive Power of FO	32
3.4	Rank- k Types	33
3.5	Proof of the Ehrenfeucht-Fraïssé Theorem	35
3.6	More Inexpressibility Results	37
3.7	Bibliographic Notes	40
3.8	Exercises	41
4	Locality and Winning Games	45
4.1	Neighborhoods, Hanf-locality, and Gaifman-locality	45
4.2	Combinatorics of Neighborhoods	49
4.3	Locality of FO	51
4.4	Structures of Small Degree	54
4.5	Locality of FO Revisited	57
4.6	Bibliographic Notes	62
4.7	Exercises	63

XII Contents

5 Ordered Structures	67
5.1 Invariant Queries	67
5.2 The Power of Order-invariant FO	69
5.3 Locality of Order-invariant FO	73
5.4 Bibliographic Notes	83
5.5 Exercises	83
6 Complexity of First-Order Logic	87
6.1 Data, Expression, and Combined Complexity	87
6.2 Circuits and FO Queries	89
6.3 Expressive Power with Arbitrary Predicates	93
6.4 Uniformity and AC ⁰	95
6.5 Combined Complexity of FO	99
6.6 Parametric Complexity and Locality	99
6.7 Conjunctive Queries	102
6.8 Bibliographic Notes	108
6.9 Exercises	109
7 Monadic Second-Order Logic and Automata	113
7.1 Second-Order Logic and Its Fragments	113
7.2 MSO Games and Types	116
7.3 Existential and Universal MSO on Graphs	119
7.4 MSO on Strings and Regular Languages	124
7.5 FO on Strings and Star-Free Languages	127
7.6 Tree Automata	129
7.7 Complexity of MSO	133
7.8 Bibliographic Notes	136
7.9 Exercises	137
8 Logics with Counting	141
8.1 Counting and Unary Quantifiers	141
8.2 An Infinitary Counting Logic	145
8.3 Games for $\mathcal{L}_{\infty\omega}^*(\mathbf{Cnt})$	151
8.4 Counting and Locality	153
8.5 Complexity of Counting Quantifiers	155
8.6 Aggregate Operators	158
8.7 Bibliographic Notes	161
8.8 Exercises	161
9 Turing Machines and Finite Models	165
9.1 Trakhtenbrot's Theorem and Failure of Completeness	165
9.2 Fagin's Theorem and NP	168
9.3 Bibliographic Notes	174
9.4 Exercises	174

10 Fixed Point Logics and Complexity Classes	177
10.1 Fixed Points of Operators on Sets	178
10.2 Fixed Point Logics	180
10.3 Properties of LFP and IFP	184
10.4 LFP, PFP, and Polynomial Time and Space	192
10.5 DATALOG and LFP	195
10.6 Transitive Closure Logic	199
10.7 A Logic for PTIME?	204
10.8 Bibliographic Notes	206
10.9 Exercises	207
11 Finite Variable Logics	211
11.1 Logics with Finitely Many Variables	211
11.2 Pebble Games	215
11.3 Definability of Types	220
11.4 Ordering of Types	225
11.5 Canonical Structures and the Abiteboul-Vianu Theorem	229
11.6 Bibliographic Notes	232
11.7 Exercises	233
12 Zero-One Laws	235
12.1 Asymptotic Probabilities and Zero-One Laws	235
12.2 Extension Axioms	238
12.3 The Random Graph	241
12.4 Zero-One Law and Second-Order Logic	243
12.5 Almost Everywhere Equivalence of Logics	245
12.6 Bibliographic Notes	246
12.7 Exercises	247
13 Embedded Finite Models	249
13.1 Embedded Finite Models: the Setting	249
13.2 Analyzing Embedded Finite Models	252
13.3 Active-Generic Collapse	256
13.4 Restricted Quantifier Collapse	260
13.5 The Random Graph and Collapse to MSO	265
13.6 An Application: Constraint Databases	267
13.7 Bibliographic Notes	270
13.8 Exercises	271
14 Other Applications of Finite Model Theory	275
14.1 Finite Model Property and Decision Problems	275
14.2 Temporal and Modal Logics	278
14.3 Constraint Satisfaction and Homomorphisms of Finite Models .	285
14.4 Bibliographic Notes	288

XIV Contents

References	291
List of Notation	305
Index	307
Name Index	313