
Preface

Numerous papers on system identification have been published over the last 40 years. Though there were substantial developments in the theory of stationary stochastic processes and multivariable statistical methods during 1950s, it is widely recognized that the theory of system identification started only in the mid-1960s with the publication of two important papers; one due to Åström and Bohlin [17], in which the maximum likelihood (ML) method was extended to a serially correlated time series to estimate ARMAX models, and the other due to Ho and Kalman [72], in which the deterministic state space realization problem was solved for the first time using a certain Hankel matrix formed in terms of impulse responses. These two papers have laid the foundation for the future developments of system identification theory and techniques [55].

The scope of the ML identification method of Åström and Bohlin [17] was to build single-input, single-output (SISO) ARMAX models from observed input-output data sequences. Since the appearance of their paper, many statistical identification techniques have been developed in the literature, most of which are now comprised under the label of *prediction error methods (PEM)* or *instrumental variable (IV) methods*. This has culminated in the publication of the volumes Ljung [109] and Söderström and Stoica [145]. At this moment we can say that theory of system identification for SISO systems is established, and the various identification algorithms have been well tested, and are now available as MATLAB[®] programs.

Also, identification of multi-input, multi-output (MIMO) systems is an important problem which is not dealt with satisfactorily by PEM methods. The identification problem based on the minimization of a prediction error criterion (or a least-squares type criterion), which in general is a complicated function of the system parameters, has to be solved by iterative descent methods which may get stuck into local minima. Moreover, optimization methods need canonical parametrizations and it may be difficult to guess a suitable canonical parametrization from the outset. Since no single continuous parametrization covers all possible multivariable linear systems with a fixed McMillan degree, it may be necessary to change parametrization in the course of the optimization routine. Thus the use of optimization criteria and canonical parametrizations can lead to local minima far from the true solution, and to

numerically ill-conditioned problems due to poor identifiability, *i.e.*, to near insensitivity of the criterion to the variations of some parameters. Hence it seems that the PEM method has inherent difficulties for MIMO systems.

On the other hand, *stochastic realization theory*, initiated by Faurre [46] and Akaike [1] and others, has brought in a different philosophy of building models from data, which is not based on optimization concepts. A key step in stochastic realization is either to apply the deterministic realization theory to a certain Hankel matrix constructed with sample estimates of the process covariances, or to apply the canonical correlation analysis (CCA) to the future and past of the observed process. These algorithms have been shown to be implemented very efficiently and in a numerically stable way by using the tools of modern numerical linear algebra such as the singular value decomposition (SVD).

Then, a new effort in digital signal processing and system identification based on the QR decomposition and the SVD emerged in the mid-1980s and many papers have been published in the literature [100, 101, 118, 119], *etc.* These realization theory-based techniques have led to a development of various so-called *subspace identification methods*, including [163, 164, 169, 171–173], *etc.* Moreover, Van Overschee and De Moor [165] have published a first comprehensive book on subspace identification of linear systems. An advantage of subspace methods is that we do not need (non-linear) optimization techniques, nor we need to impose to the system a canonical form, so that subspace methods do not suffer from the inconveniences encountered in applying PEM methods to MIMO system identification.

Though I have been interested in stochastic realization theory for many years, it was around 1990 that I actually resumed studies on realization theory, including subspace identification methods. However, realization results developed for deterministic systems on the one hand, and stochastic systems on the other, could not be applied to the identification of dynamic systems in which both a deterministic test input and a stochastic disturbance are involved. In fact, the deterministic realization result does not consider any noise, and the stochastic realization theory developed up to the early 1990s did address modeling of stochastic processes, or time series, only. Then, I noticed at once that we needed a new realization theory to understand many existing subspace methods and their underlying relations and to develop advanced algorithms. Thus I was fully convinced that a new stochastic realization theory in the presence of exogenous inputs was needed for further developments of subspace system identification theory and algorithms.

While we were attending the MTNS (The International Symposium on Mathematical Theory of Networks and Systems) at Regensburg in 1993, I suggested to Giorgio Picci, University of Padova, that we should do joint work on stochastic realization theory in the presence of exogenous inputs and a collaboration between us started in 1994 when he stayed at Kyoto University as a visiting professor. Also, I successively visited him at the University of Padova in 1997. The collaboration has resulted in several joint papers [87–90, 93, 130, 131]. Professor Picci has in particular introduced the idea of decomposing the output process into deterministic and stochastic components by using a preliminary orthogonal decomposition, and then applying the existing deterministic and stochastic realization techniques to each com-

ponent to get a realization theory in the presence of exogenous input. On the other hand, inspired by the CCA-based approach, I have developed a method of solving a multi-stage Wiener prediction problem to derive an innovation representation of the stationary process with an observable exogenous input, from which subspace identification methods are successfully obtained.

This book is an outgrowth of the joint work with Professor Picci on stochastic realization theory and subspace identification. It provides an in-depth introduction to subspace methods for system identification of discrete-time linear systems, together with our results on realization theory in the presence of exogenous inputs and subspace system identification methods. I have included proofs of theorems and lemmas as much as possible, as well as solutions to problems, in order to facilitate the basic understanding of the material by the readers and to minimize the effort needed to consult many references.

This textbook is divided into three parts: Part I includes reviews of basic results, from numerical linear algebra to Kalman filtering, to be used throughout this book, Part II provides deterministic and stochastic realization theories developed by Ho and Kalman, Faurre, and Akaike, and Part III discusses stochastic realization results in the presence of exogenous inputs and their adaptation to subspace identification methods; see Section 1.6 for more details. Thus, various people can read this book according to their needs. For example, people with a good knowledge of linear system theory and Kalman filtering can begin with Part II. Also, people mainly interested in applications can just read the algorithms of the various identification methods in Part III, occasionally returning to Part I and/or Part II when needed. I believe that this textbook should be suitable for advanced students, applied scientists and engineers who want to acquire solid knowledge and algorithms of subspace identification methods.

I would like to express my sincere thanks to Giorgio Picci who has greatly contributed to our fruitful collaboration on stochastic realization theory and subspace identification methods over the last ten years. I am deeply grateful to Hideaki Sakai, who has read the whole manuscript carefully and provided invaluable suggestions, which have led to many changes in the manuscript. I am also grateful to Kiyotsugu Takaba and Hideyuki Tanaka for their useful comments on the manuscript. I have benefited from joint works with Takahira Ohki, Toshiaki Itoh, Morimasa Ogawa, and Hajime Ase, who told me about many problems regarding modeling and identification of industrial processes.

The related research from 1996 through 2004 has been sponsored by the Grant-in-Aid for Scientific Research, the Japan Society of Promotion of Sciences, which is gratefully acknowledged.

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January 2005*

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