

Contents

Part I Background

1	Background on Nonlinear Systems	3
1.1	Preliminaries	3
1.2	Families of Nonlinear Systems	4
1.2.1	Nonautonomous Systems	5
1.2.2	Autonomous Systems	6
1.2.3	Systems with Inputs	6
1.2.4	Discrete Time Dynamics	7
1.3	Notions of Stability	8
1.3.1	Stability	9
1.3.2	Asymptotic and Exponential Stability	9
1.3.3	Input-to-State Stability	10
1.3.4	Linear Systems and Linearizations	12
1.3.5	Uniformity vs. Non-uniformity	12
1.3.6	Basin of Attraction	13
1.4	Stabilization	13
1.5	Examples	15
1.5.1	Stable System	15
1.5.2	Locally Asymptotically Stable System	16
1.5.3	Globally Asymptotically Stable System	17
1.5.4	UGAS Time-Varying System	18
1.5.5	Systems in Chained Form	19
1.6	Comments	20
2	Review of Lyapunov Functions	25
2.1	Strict Lyapunov Function	25
2.1.1	Definition	25
2.1.2	Lemmas	27
2.1.3	ISS Lyapunov Function	29
2.2	Non-strict Lyapunov Function	31

2.2.1	Matrosov Theorems	32
2.2.2	LaSalle Invariance Principle	33
2.2.3	Jurdjevic-Quinn Theorem	34
2.3	Discrete Time Lyapunov Function	35
2.4	Illustrations	36
2.4.1	Strict Lyapunov Function	36
2.4.2	ISS Lyapunov Function	37
2.4.3	iISS Lyapunov Function	39
2.4.4	LaSalle Invariance Principle	40
2.4.5	Matrosov Theorems	40
2.4.6	Non-strict Lyapunov-Like Function	41
2.5	Basin of Attraction Revisited	44
2.6	\mathcal{L}_2 Gains	45
2.6.1	Basic Theorem	45
2.6.2	Illustration	46
2.7	Lyapunov Functions with Bounded Gradients	47
2.7.1	Effect of Exponentially Decaying Disturbances	48
2.7.2	Dependence on Coordinates	50
2.7.3	Strictification	51
2.8	Comments	53

Part II Time-Invariant Case

3	Matrosov Conditions: Simple Case	61
3.1	Motivation	61
3.2	Continuous Time Theorem	64
3.3	Proof of Continuous Time Theorem	66
3.4	Discrete Time Theorem	69
3.5	Proof of Discrete Time Theorem	71
3.6	Illustrations	75
3.6.1	Continuous Time: One Auxiliary Function	75
3.6.2	Continuous Time: Two Auxiliary Functions	76
3.6.3	Discrete Time Context	79
3.7	Comments	80
4	Jurdjevic-Quinn Conditions	83
4.1	Motivation	83
4.2	Control Affine Case	85
4.2.1	Assumptions and Statement of Result	85
4.2.2	Main Lemmas	86
4.2.3	Checking the CLF Properties	88
4.2.4	Arbitrarily Small Stabilizing Feedbacks	91
4.3	General Case	93
4.4	Construction of the Auxiliary Scalar Field	96
4.5	Hamiltonian Systems	98

4.6	Robustness	102
4.7	Illustrations	107
4.7.1	Two-Dimensional Example	108
4.7.2	Two-Link Manipulator Revisited	109
4.8	Comments	114
5	Systems Satisfying the Conditions of LaSalle	117
5.1	Background and Motivation	117
5.2	First Method: Iterated Lie Derivatives	118
5.3	Discussion and Extensions of First Method	124
5.3.1	Local vs. Global	124
5.3.2	Real Analytic Case	124
5.3.3	Necessity vs. Sufficiency	125
5.3.4	Recovering Exponential Stability	125
5.4	Second Method: Matrosov Conditions	126
5.5	Application: Lotka-Volterra Model	129
5.5.1	Strict Lyapunov Function Construction	129
5.5.2	Robustness to Uncertainty	132
5.5.3	Numerical Validation	134
5.6	Comments	136

Part III Time-Varying Case

6	Strictification: Basic Results	141
6.1	Background	141
6.2	Motivating Example	143
6.3	Time-Varying Strictification Theorem	145
6.4	Remarks on Rate of Convergence	150
6.5	Input-to-State Stability	152
6.6	Equivalent Characterizations of Non-strict ISS	155
6.6.1	Proofs of Equivalences	155
6.6.2	Remarks on Proof of Equivalences	157
6.7	Input-to-Output Stability	158
6.8	Illustrations	160
6.8.1	Rotating Rigid Body	160
6.8.2	Stabilization of Underactuated Ships	162
6.9	Comments	171
7	Backstepping for Time-Varying Systems	175
7.1	Motivation: PVTOL	175
7.2	Classical Backstepping	177
7.2.1	Backstepping with Cancelation	177
7.2.2	Backstepping with Domination	180
7.2.3	Further Extensions	185
7.3	Backstepping for Nonautonomous Systems	186

7.3.1	Chained Form Systems	187
7.3.2	Feedback Systems	189
7.3.3	Feedforward Systems	190
7.3.4	Other Important Cases	191
7.4	Linear Time-Varying Systems	192
7.4.1	General Result for Linear Time-Varying Systems	193
7.4.2	Linear Time-Varying Systems in Feedback Form	199
7.4.3	Illustration: Linear System with PE Coefficients	205
7.5	Nonlinear Time-Varying Systems	210
7.5.1	Assumptions and Notation	210
7.5.2	Main Result and Remarks	211
7.6	Bounded Backstepping	213
7.6.1	Assumptions and Statement of Result	213
7.6.2	Technical Lemmas	214
7.6.3	Proof of Bounded Backstepping Theorem	217
7.7	Two-Dimensional Example	219
7.8	PVTOL Revisited	221
7.8.1	Analysis of Reduced System	222
7.8.2	UGAS of Full System	226
7.8.3	Numerical Example	227
7.9	Comments	227
8	Matrosov Conditions: General Case	231
8.1	Motivation	231
8.2	Preliminaries and Matrosov Assumptions	233
8.3	One Auxiliary Function	235
8.4	Several Auxiliary Functions	238
8.5	Persistency of Excitation	240
8.6	Applications	242
8.6.1	Jurdjevic-Quinn Theorems	242
8.6.2	LaSalle Type Conditions	244
8.7	Sign Constrained Controller	245
8.7.1	Verifying the Assumptions of the Theorem	246
8.7.2	Strict Lyapunov Function Construction	247
8.8	Comments	252
9	Adaptively Controlled Systems	253
9.1	Overview of Adaptive Control	253
9.2	Motivating Example	254
9.3	Assumptions and Main Construction	256
9.4	Robustness	260
9.4.1	Statement of ISS Theorem	260
9.4.2	Proof of ISS Theorem	261
9.5	Rössler System Revisited	264
9.6	Lorenz System	265

9.7 Extension: More General Feedbacks	268
9.8 Comments	271

Part IV Systems with Multiple Time Scales

10 Rapidly Time-Varying Systems	275
10.1 Motivation	275
10.2 Overview of Methods	276
10.3 Assumptions and Lemmas	277
10.4 Main Lyapunov Function Construction	279
10.5 Alternative Strictification Result	284
10.6 Illustrations	286
10.6.1 A UGAS Dynamics that Is Not UGES	287
10.6.2 System Arising in Identification	287
10.6.3 Friction Example Revisited	290
10.7 Further Illustrations: Strictification Approach	292
10.7.1 Systems with Unknown Functional Parameters	292
10.7.2 Dynamics that Are Not Globally Lipschitz	293
10.8 Comments	293
11 Slowly Time-Varying Systems	297
11.1 Motivation	297
11.2 Overview of Methods	298
11.3 Assumptions and Lemmas	299
11.4 Main Lyapunov Construction	299
11.5 More General Decay Rates	301
11.6 Illustrations	302
11.6.1 A Scalar Example	303
11.6.2 Pendulum Example Revisited	304
11.6.3 Friction Example Revisited	305
11.6.4 Identification Dynamics Revisited	307
11.7 Input-to-State Stability	310
11.8 Comments	312
12 Hybrid Time-Varying Systems	317
12.1 Motivation	317
12.2 Preliminaries	320
12.2.1 ISS and PE in Continuous and Discrete Time	320
12.2.2 Hybrid Systems	323
12.3 Strictification for Time-Varying Systems	327
12.3.1 Discrete Time Strictification	327
12.3.2 Hybrid Strictification	329
12.4 Matrosov Constructions for Time-Varying Systems	331
12.4.1 Discrete Time Construction	332
12.4.2 Hybrid Version	336

12.5 Illustrations	338
12.6 Comments	340
Part V Appendices	
A Some Lemmas	345
A.1 Useful Families of Functions	345
A.2 Some Useful Inequalities.....	354
A.3 A Lower Bound for the Lotka-Volterra Model	356
A.4 ISS and iISS for the Lotka-Volterra Model	358
A.5 Useful Integral	361
A.6 Continuous Time Matrosov Result with PE	364
B Converse Theory	367
B.1 Converse Lyapunov Function Theorem	367
B.2 Time-Varying Converse ISS Result.....	370
References	373
Index	383