

Contents

1	What is a Computable Model?	1
1.1	Mathematical Models	1
1.2	Specifications, Programs, and Models	2
1.3	Data Types and Programming Languages	3
1.4	Theories of Data	4
1.5	Recursive Models	5
1.6	Intensional Models	6
1.7	A Logical Foundation for Specification	7
1.8	Implementable Models	7
1.9	The Logical Setting	8
	References	9
2	Typed Predicate Logic	11
2.1	Judgments and Contexts	11
2.2	Structural Rules	13
2.3	Types	13
2.4	Relations and Functions	15
2.5	Equality	15
2.6	Propositional Rules	16
2.7	Quantifier Rules	17
2.8	TPL Derivations	18
2.9	Type Inference	20
	References	23
3	Data Types	25
3.1	Booleans	25
3.2	Products	27
3.3	Stacks	29
3.4	Terms	30
3.5	Numbers	30
3.6	Lists	32
3.7	A Type of Types	33

- 3.8 Theories of Data Types 33
- References 35
- 4 Definability** 37
 - 4.1 Semidecidable Relations 38
 - 4.2 Decidable Relations 39
 - References 41
- 5 Specification** 43
 - 5.1 A Logical Perspective 44
 - 5.2 Some Specifications 45
 - 5.3 Operations on Schema 48
 - 5.4 Conservative Extensions 51
 - References 52
- 6 Functions** 53
 - 6.1 Totality and Functionality 53
 - 6.2 Functional Application 54
 - 6.3 Explicit Functions 57
 - 6.4 The Elimination of Application 59
 - References 62
- 7 Preconditions** 63
 - 7.1 Specifications with Preconditions 63
 - 7.2 Totality and Functionality 65
 - 7.3 Functional Application 67
 - 7.4 Application Elimination 69
 - 7.5 Partial Functions 69
 - References 70
- 8 Natural Numbers** 71
 - 8.1 A Theory of Numbers 71
 - 8.2 Numerical Specification 74
 - 8.3 Recursive Specifications 79
 - 8.4 Enriched Arithmetic 82
 - 8.5 Arithmetic Interpretation 83
 - References 84
- 9 Typed Set Theory** 85
 - 9.1 CST 85
 - 9.2 Elementary Properties 88
 - 9.3 Subsets and Extensionality 89
 - 9.4 New Sets from Old 90
 - 9.5 Set-Theoretic Relations 97

- 9.6 Arithmetic Interpretation 100
- References 102
- 10 Systems Modeling** 103
 - 10.1 The Requirements 103
 - 10.2 The State 104
 - 10.3 Operations 107
 - 10.4 A Mathematical Model 108
 - References 111
- 11 A Type of Types** 113
 - 11.1 The Type type 114
 - 11.2 Dependent Types 115
 - 11.3 Dependent Specifications 116
 - 11.4 Polymorphic Specifications 117
 - 11.5 Polymorphic Set Theory 120
 - 11.6 Specifications and Types 122
 - 11.7 Arithmetic Interpretation 124
 - References 125
- 12 Schemata** 127
 - 12.1 A Theory of Relations 127
 - 12.2 A Minimal Theory 130
 - 12.3 Operations on Schemata 132
 - 12.4 Arithmetic Interpretation 139
 - References 140
- 13 Separation Types** 143
 - 13.1 Theories with Separation 143
 - 13.2 Subtypes in Specification 145
 - 13.3 Preconditions and Functions 146
 - 13.4 Polymorphism and Subtypes 148
 - 13.5 The Elimination of Subtypes 149
 - References 153
- 14 Recursive Schemata** 155
 - 14.1 Closure and Induction 155
 - 14.2 Simultaneous Recursion 158
 - 14.3 Arithmetic Interpretation 162
 - 14.4 Sets and Schemata 162
 - References 166
- 15 Inductive Types** 167
 - 15.1 The General Form 167

15.2	Some Inductive Types	168
15.3	Conservative Extensions	170
15.4	Finite Schemata	171
	References	175
16	Recursive Functions	177
16.1	General Form	177
16.2	Numerical Recursion	180
16.3	Recursive Functions and Inductive Types	181
	References	183
17	Schema Definitions	185
17.1	Schema Definitions	185
17.2	Refinement	188
17.3	Implementable Definitions	191
17.4	The Limits of Refinement	192
17.5	Properties of Schemata	193
	References	194
18	Computable Ontology	195
18.1	Implementable Models	195
18.2	A Type of Events	196
18.3	Arithmetic Interpretation	197
18.4	Instances	198
18.5	Implementation	199
	References	199
19	Classes	201
19.1	Classes and Judgments	201
19.2	Class Elimination	204
	References	205
20	Classes of Functions	207
20.1	Function Application	207
20.2	Specifications and Function Classes	209
20.3	Partial Functions	212
20.4	Polymorphism	213
	References	215
21	Computable Analysis	217
21.1	Cauchy Sequences	217
21.2	Operations on the Real Numbers	219
21.3	Implementation	220
	References	222

- 22 Programming Language Specification** 223
 - 22.1 The Abstract Machine 223
 - 22.2 A Programming Language and Its Specification 226
 - 22.3 Implementation 228
 - References 229

- 23 Abstract Types** 231
 - 23.1 Axiomatic Specifications 231
 - 23.2 Polymorphism and Data Abstraction 234
 - References 236

- 24 Conclusion** 237

- Index** 239