Contents

1. Introduction 1
- 1.1 Overview 1
- 1.2 What this Book is About 1
- 1.3 What this Book Tries to Do 6
- 1.4 What this Book Tries Not to Do 6
- 1.5 The Exercises 7
- 1.6 Further Reading 7
- 1.7 Some Advice 7

Part 1 Language and Machines

2. Elements of Formal Languages 11
- 2.1 Overview 11
- 2.2 Alphabets 11
- 2.3 Strings 12
  - 2.3.1 Functions that Apply to Strings 12
  - 2.3.2 Useful Notation for Describing Strings 13
- 2.4 Formal Languages 15
- 2.5 Methods for Defining Formal Languages 16
  - 2.5.1 Set Definitions of Languages 17
  - 2.5.2 Decision Programs for Languages 19
  - 2.5.3 Rules for Generating Languages 20
- 2.6 Formal Grammars 25
  - 2.6.1 Grammars, Derivations and Languages 26
  - 2.6.2 The Relationship between Grammars and Languages 29

2.7 Phrase Structure Grammars and the Chomsky Hierarchy 30
2.7.1 Formal Definition of Phrase Structure Grammars 30
2.7.2 Derivations, Sentential Forms, Sentences and "L(G)" 31
2.7.3 The Chomsky Hierarchy 35
2.8 A Type 0 Grammar: Computation as Symbol Manipulation 38

3. Syntax, Semantics and Ambiguity 43
3.1 Overview 43
3.2 Syntax vs. Semantics 43
3.3 Derivation Trees 44
3.4 Parsing 47
3.5 Ambiguity 49

4. Regular Languages and Finite State Recognisers 55
4.1 Overview 55
4.2 Regular Grammars 55
4.3 Some Problems with Grammars 57
4.4 Finite State Recognisers and Finite State Generators 58
4.4.1 Creating an FSR 58
4.4.2 The Behaviour of the FSR 60
4.4.3 The FSR as Equivalent to the Regular Grammar 64
4.5 Non-determinism in Finite State Recognisers 67
4.5.1 Constructing Deterministic FSRs 69
4.5.2 The Deterministic FSR as Equivalent to the Non-deterministic FSR 72
4.6 A Simple Deterministic Decision Program 77
4.7 Minimal FSRs 77
4.7.1 Constructing a Minimal FSR 79
4.7.2 Why Minimisation Works 83
4.8 The General Equivalence of Regular Languages and FSRs 86
4.9 Observations on Regular Grammars and Languages 88

5. Context Free Languages and Pushdown Recognisers 93
5.1 Overview 93
5.2 Context Free Grammars and Context Free Languages 94
5.3 Changing **G** without Changing $L(G)$ 94
5.3.1 The Empty String (ε) 95
5.3.2 Chomsky Normal Form 98
5.4 Pushdown Recognisers 104
5.4.1 The Stack 105
5.4.2 Constructing a Non-deterministic PDR 105
5.4.3 Example NPDRs, M_3 and M_{10} 107

5.5 Deterministic Pushdown Recognisers 112
5.5.1 $M_3{}^d$, a Deterministic Version of M_3 113
5.5.2 More Deterministic PDRs 115
5.6 Deterministic and Non-deterministic Context Free Languages 116
5.6.1 Every Regular Language is a Deterministic CFL 116
5.6.2 The Non-deterministic CFLs 118
5.6.3 A Refinement to the Chomsky Hierarchy in the Case of CFLs 120
5.7 The Equivalence of CFGs and PDRs 121
5.8 Observations on Context Free Grammars and Languages 121

6. Important Features of Regular and Context Free Languages 125
6.1 Overview 125
6.2 Closure Properties of Languages 126
6.3 Closure Properties of the Regular Languages 126
6.3.1 Complement 126
6.3.2 Union 128
6.3.3 Intersection 129
6.3.4 Concatenation 131
6.4 Closure Properties of the Context Free Languages 133
6.4.1 Union 133
6.4.2 Concatenation 135
6.4.3 Intersection 136
6.4.4 Complement 137
6.5 Chomsky's Hierarchy is Indeed a Proper Hierarchy 138
6.5.1 The "Repeat State Theorem" 139
6.5.2 A Language that is Context Free but not Regular 142
6.5.3 The "uvwxy" Theorem for Context Free Languages 143
6.5.4 $\{a^i b^i c^i : i \geq 1\}$ is not Context Free 150
6.5.5 The "Multiplication Language" is not Context Free 151
6.6 Preliminary Observations on the Scope of the Chomsky Hierarchy 153

7. Phrase Structure Languages and Turing Machines 155
7.1 Overview 155
7.2 The Architecture of the Turing Machine 155
7.2.1 "Tapes" and the "Read/Write Head" 156
7.2.2 Blank Squares 157
7.2.3 TM "Instructions" 158
7.2.4 Turing Machines Defined 159
7.3 The Behaviour of a TM 159

7.4 Turing Machines as Language Recognisers 163
7.4.1 Regular Languages 163
7.4.2 Context Free Languages 164
7.4.3 Turing Machines are More Powerful than PDRs 166
7.5 Introduction to (Turing Machine) Computable Languages 169
7.6 The TM as the Recogniser for the Context Sensitive Languages 170
7.6.1 Constructing a Non-deterministic TM for Reduction Parsing of a Context Sensitive Language 171
7.6.2 The Generality of the Construction 175
7.7 The TM as the Recogniser for the Type 0 Languages 177
7.7.1 Amending the Reduction Parsing TM to Deal with Type 0 Productions 178
7.7.2 Dealing with the Empty String 178
7.7.3 The TM as the Recogniser for all Types in the Chomsky Hierarchy 181
7.8 Decidability: A Preliminary Discussion 181
7.8.1 Deciding a Language 181
7.8.2 Accepting a Language 183
7.9 End of Part One 184

Part 2 Machines and Computation

8. Finite State Transducers 189
8.1 Overview. 189
8.2 Finite State Transducers 189
8.3 Finite State Transducers and Language Recognition 190
8.4 Finite State Transducers and Memory. 191
8.5 Finite State Transducers and Computation 194
8.5.1 Simple Multiplication 194
8.5.2 Addition and Subtraction 195
8.5.3 Simple Division and Modular Arithmetic 199
8.6 The Limitations of the Finite State Transducer 200
8.6.1 Restricted FST Multiplication 201
8.6.2 FSTs and Unlimited Multiplication 204
8.7 FSTs as Unsuitable Models for Real Computers 204

9. Turing Machines as Computers 209
9.1 Overview. 209
9.2 Turing Machines and Computation. 209
9.3 Turing Machines and Arbitrary Binary Multiplication. 210

9.3.1 Some Basic TM Operations 210
9.3.2 The "ADD" TM. 212
9.3.3 The "MULT" TM 215
9.4 Turing Machines and Arbitrary Integer Division 222
9.4.1 The "SUBTRACT" TM 222
9.4.2 The "DIV" TM 225
9.5 Logical Operations 226
9.6 TMs and the Simulation of Computer Operations. 229

10. Turing's Thesis and the Universality of the Turing Machine . . . 237
10.1 Overview. 237
10.2 Turing's Thesis 238
10.3 Coding a Turing Machine and its Tape as a Binary Number 240
10.3.1 Coding Any TM. 241
10.3.2 Coding the Tape 244
10.4 The Universal Turing Machine 244
10.4.1 *UTM*'s Tapes 245
10.4.2 The Operation of *UTM* 246
10.4.3 Some Implications of *UTM* 249
10.5 Non-deterministic TMs. 249
10.6 Converting a Non-deterministic TM into a 4-tape Deterministic TM 250
10.6.1 The Four Tapes of the Deterministic Machine, *D*. 251
10.6.2 The Systematic Generation of the Strings of Quintuple Labels 253
10.6.3 The Operation of *D* 257
10.6.4 The Equivalence of Non-deterministic TMs and 4-Tape Deterministic TMs 260
10.7 Converting a Multi-tape TM into a Single-tape TM 261
10.7.1 Example: Representing Three Tapes as One. 263
10.7.2 The Operation of the Single-tape Machine, *S* 265
10.7.3 The Equivalence of Deterministic Multi-tape TMs and Deterministic Single-tape TMs. 266
10.8 The Linguistic Implications of the Equivalence of Non-deterministic and Deterministic TMs. 267

11. Computability, Solvability and the Halting Problem 269
11.1 Overview. 269
11.2 The Relationship Between Functions, Problems, Solvability and Decidability 270
11.2.1 Functions and Computability 270
11.2.2 Problems and Solvability. 271
11.2.3 Decision Problems and Decidability 272

11.3 The Halting Problem 273
11.3.1 UTM_H Partially Solves the Halting Problem 274
11.3.2 *Reductio ad Absurdum* Applied to the Halting Problem 275
11.3.3 The halting problem shown to be unsolvable 277
11.3.4 Some Implications of the Unsolvability of the Halting Problem 280
11.4 Computable Languages 282
11.4.1 An Unacceptable (non-Computable) Language 283
11.4.2 An Acceptable, But Undecidable, Language. 285
11.5 Languages and Machines 286

12. Dimensions of Computation 291
12.1 Overview. 291
12.2 Aspects of Computation: Space, Time and Complexity 292
12.3 Non-Deterministic TMs Viewed as Parallel Processors 294
12.3.1 Parallel Computations and Time 296
12.4 A Brief Look at an Unsolved Problem of Complexity 298
12.5 A Beginner's Guide to the "Big O" 298
12.5.1 Predicting the Running Time of Algorithms. 298
12.5.2 Linear time. 300
12.5.3 Logarithmic Time 302
12.5.4 Polynomial Time 305
12.5.5 Exponential Time 313
12.6 The Implications of Exponential Time Processes. 315
12.6.1 "Is P Equal to NP?". 316
12.7 Observations on the Efficiency of Algorithms 317

Further Reading 319

Solutions to Selected Exercises 323
Chapter 2 323
Chapter 3 325
Chapter 4 326
Chapter 5 328
Chapter 6 329
Chapter 7 330
Chapter 8 330
Chapter 9 330
Chapter 10 331
Chapter 11 331
Chapter 12 331

Index 335