

CONTENTS

PREFACE	vii
CHAPTER 1. INTRODUCTION	1
Problems for Chapter 1, 5.	
CHAPTER 2. EQUINUMEROSITY	7
Countable unions of countable sets, 9. The reals are uncountable, 11. $A <_c \mathcal{P}(A)$, 14. Schröder-Bernstein Theorem, 16. Problems for Chapter 2, 17.	
CHAPTER 3. PARADOXES AND AXIOMS	19
The Russell paradox, 21. Axioms (I) – (VI), 24. Axioms for definite conditions and operations, 26. Classes, 27. Problems for Chapter 3, 30.	
CHAPTER 4. ARE SETS ALL THERE IS?	33
Ordered pairs, 34. Disjoint union, 35. Relations, 36. Equivalence relations, 37. Functions, 38. Cardinal numbers, 42. Structured sets, 44. Problems for Chapter 4, 45.	
CHAPTER 5. THE NATURAL NUMBERS	51
Peano systems, 51. Existence of the natural numbers, 52. Uniqueness of the natural numbers, 52. Recursion Theorem, 53. Addition and multiplication, 58. Pigeonhole Principle, 62. Strings, 64. String recursion, 66. The continuum, 67. Problems for Chapter 5, 67.	
CHAPTER 6. FIXED POINTS	71
Posets, 71. Partial functions, 74. Inductive posets, 75. Continuous Least Fixed Point Theorem, 76. About topology, 79. Graphs, 82. Problems for Chapter 6, 83. Streams, 84. Scott topology, 87. Directed-complete posets, 88.	
CHAPTER 7. WELL ORDERED SETS	89
Transfinite induction, 94. Transfinite recursion, 95. Iteration Lemma, 96. Compa- rability of well ordered sets, 99. Wellfoundedness of \leq_o , 100. Hartogs' Theorem, 100. Fixed Point Theorem, 102. Least Fixed Point Theorem, 102. Problems for Chapter 7, 104.	

CHAPTER 8. CHOICES	109
Axiom of Choice, 109. Equivalents of AC , 112. Maximal Chain Principle, 114. Zorn's Lemma, 114. Countable Principle of Choice, \mathbf{AC}_{\aleph} , 114. Axiom (VII) of Dependent Choices, DC , 114. The axiomatic theory ZDC , 117. Consistency and independence results, 117. Problems for Chapter 8, 119.	
CHAPTER 9. CHOICE'S CONSEQUENCES	121
Trees, 122. König's Lemma, 123. Fan Theorem, 123. Well foundedness of \leq_c , 124. Best wellorderings, 124. König's Theorem, 128. Cofinality, regular and singular cardinals, 129. Problems for Chapter 9, 130.	
CHAPTER 10. BAIRE SPACE	135
Cardinality of perfect pointsets, 138. Cantor-Bendixson Theorem, 139. Property P , 140. Analytic pointsets, 141. Perfect Set Theorem, 144. Borel sets, 147. The Separation Theorem, 149. Suslin's Theorem, 150. Counterexample to the general property P , 150. Consistency and independence results, 152. Problems for Chapter 10, 153. Borel isomorphisms, 154.	
CHAPTER 11. REPLACEMENT AND OTHER AXIOMS	157
Replacement Axiom (VIII), 158. The theory ZFDC , 158. Grounded Recursion Theorem, 159. Transitive classes, 161. Basic Closure Lemma, 162. The grounded, pure, hereditarily finite sets, 163. Zermelo universes, 164. The least Zermelo universe, 165. Grounded sets, 166. Principle of Foundation, 167. The theory ZFC (Zermelo-Fraenkel with choice), 167. ZFDC -universes, 169. von Neumann's class \mathcal{V} , 169. Mostowski Collapsing Lemma, 170. Consistency and independence results, 171. Problems for Chapter 11, 171.	
CHAPTER 12. ORDINAL NUMBERS	175
Ordinal numbers, 176. The least infinite ordinal ω , 177. Characterization of ordinal numbers, 179. Ordinal recursion, 182. Ordinal addition and multiplication, 183. von Neumann cardinals, 184. The operation \aleph_α , 186. The cumulative rank hierarchy, 187. Problems for Chapter 12, 190. The operation \beth_α , 194. Strongly inaccessible cardinals, 195. Frege cardinals, 196. Quotients of equivalence conditions, 197.	
APPENDIX A. THE REAL NUMBERS	199
Congruences, 199. Fields, 201. Ordered fields, 202. Existence of the rationals, 204. Countable, dense, linear orderings, 208. The archimedean property, 210. Nested interval property, 213. Dedekind cuts, 216. Existence of the real numbers, 217. Uniqueness of the real numbers, 220. Problems for Appendix A, 222.	
APPENDIX B. AXIOMS AND UNIVERSES	225
Set universes, 228. Propositions and relativizations, 229. Rieger universes, 232. Rieger's Theorem, 233. Antifoundation Principle, AFA , 238. Bisimulations, 239. The antifounded universe, 242. Aczel's Theorem, 243. Problems for Appendix B, 245.	
SOLUTIONS TO THE EXERCISES IN CHAPTERS 1 – 12	249
INDEX	271