
Contents

Notation	xxi
1 Introduction	1
1.1 References.....	5
2 Probability and Statistical Models	7
2.1 Introduction.....	7
2.2 Axioms of Probability.....	7
2.2.1 Independence.....	8
2.2.2 Bayes' law.....	8
2.3 Probability Distributions.....	9
2.3.1 Random variables.....	9
2.3.2 Independence.....	10
2.3.3 Cumulative distribution functions.....	10
2.3.4 Quantiles and percentiles.....	11
2.3.5 Expectations and variances.....	12
2.3.6 Does the expected value exist?.....	13
2.4 Functions of Random Variables.....	14
2.5 Random Samples.....	15
2.6 The Binomial Distribution.....	16
2.7 Location, Scale, and Shape Parameters.....	17
2.8 Some Common Continuous Distributions.....	17
2.8.1 Uniform distributions.....	17
2.8.2 Normal distributions.....	18
2.8.3 The lognormal distribution.....	20
2.8.4 Exponential and double exponential distributions.....	21
2.9 Sampling a Normal Distribution.....	21
2.9.1 Chi-squared distributions.....	21
2.9.2 t -distributions.....	22
2.9.3 F -distributions.....	22
2.10 Order Statistics and the Sample CDF.....	23

2.10.1	Normal probability plots	23
2.11	Skewness and Kurtosis	24
2.12	Heavy-Tailed Distributions	28
2.12.1	Double exponential distributions	30
2.12.2	t -distributions have heavy tails	30
2.12.3	Mixture models	31
2.12.4	Pareto distributions	32
2.12.5	Distributions with Pareto tails	34
2.13	Law of Large Numbers and Central Limit Theorem	36
2.14	Multivariate Distributions	37
2.14.1	Correlation and covariance	38
2.14.2	Independence and covariance	40
2.14.3	The multivariate normal distribution	41
2.15	Prediction	42
2.15.1	Best linear prediction	42
2.15.2	Prediction error in linear prediction	43
2.15.3	Multivariate linear prediction	44
2.16	Conditional Distributions	44
2.16.1	Best prediction	45
2.16.2	Normal distributions: Conditional expectations and variance	45
2.17	Linear Functions of Random Variables	46
2.17.1	Two linear combinations of random variables	48
2.17.2	Independence and variances of sums	49
2.17.3	Application to normal distributions	49
2.18	Estimation	49
2.18.1	Maximum likelihood estimation	50
2.18.2	Standard errors	52
2.18.3	Fisher information	53
2.18.4	Bayes estimation*	54
2.18.5	Robust estimation*	56
2.19	Confidence Intervals	60
2.19.1	Confidence interval for the mean	60
2.19.2	Confidence intervals for the variance and standard deviation	61
2.19.3	Confidence intervals based on standard errors	62
2.20	Hypothesis Testing	62
2.20.1	Hypotheses, types of errors, and rejection regions	62
2.20.2	P -values	63
2.20.3	Two-sample t -tests	64
2.20.4	Statistical versus practical significance	65
2.20.5	Tests of normality	66
2.20.6	Likelihood ratio tests	66
2.21	Summary	68
2.22	Bibliographic Notes	70

2.23	References	71
2.24	Problems	72
3	Returns	75
3.1	Introduction	75
3.1.1	Net returns	75
3.1.2	Gross returns	75
3.1.3	Log returns	76
3.1.4	Adjustment for dividends	77
3.2	Behavior Of Returns	78
3.3	The Random Walk Model	80
3.3.1	I.i.d. normal returns	80
3.3.2	The lognormal model	80
3.3.3	Random walks	82
3.3.4	Geometric random walks	82
3.3.5	The effect of the drift μ	83
3.3.6	Are log returns normally distributed?	84
3.3.7	Do the GE daily returns look like a geometric random walk?	86
3.4	Origins of the Random Walk Hypothesis	89
3.4.1	Fundamental analysis	89
3.4.2	Technical analysis	91
3.5	Efficient Markets Hypothesis (EMH)	93
3.5.1	Three types of efficiency	93
3.5.2	Testing market efficiency	94
3.6	Discrete and Continuous Compounding	95
3.7	Summary	96
3.8	Bibliographic Notes	97
3.9	References	97
3.10	Problems	98
4	Time Series Models	101
4.1	Time Series Data	101
4.2	Stationary Processes	102
4.2.1	Weak white noise	103
4.2.2	Predicting white noise	104
4.2.3	Estimating parameters of a stationary process	104
4.3	AR(1) Processes	105
4.3.1	Properties of a stationary AR(1) process	106
4.3.2	Convergence to the stationary distribution	108
4.3.3	Nonstationary AR(1) processes	108
4.4	Estimation of AR(1) Processes	110
4.4.1	Residuals and model checking	111
4.4.2	AR(1) model for GE daily log returns	113
4.5	AR(p) Models	115

4.5.1	AR(6) model for GE daily log returns	117
4.6	Moving Average (MA) Processes	118
4.6.1	MA(1) processes	118
4.6.2	General MA processes	118
4.6.3	MA(2) model for GE daily log returns	119
4.7	ARIMA Processes	120
4.7.1	The backwards operator	120
4.7.2	ARMA processes	120
4.7.3	Fitting ARMA processes: GE daily log returns	121
4.7.4	The differencing operator	122
4.7.5	From ARMA processes to ARIMA processes	122
4.7.6	ARIMA(2,1,0) model for GE daily log prices	123
4.8	Model Selection	124
4.8.1	AIC and SBC	124
4.8.2	GE daily log returns: Choosing the AR order	125
4.9	Three-Month Treasury Bill Rates	126
4.10	Forecasting	128
4.10.1	Forecasting GE daily log returns and log prices	130
4.11	Summary	131
4.12	Bibliographic Notes	134
4.13	References	134
4.14	Problems	134
5	Portfolio Theory	137
5.1	Trading Off Expected Return and Risk	137
5.2	One Risky Asset and One Risk-Free Asset	137
5.2.1	Estimating $E(R)$ and σ_R	140
5.3	Two Risky Assets	140
5.3.1	Risk versus expected return	140
5.3.2	Estimating means, standard deviations, and covariances	141
5.4	Combining Two Risky Assets with a Risk-Free Asset	142
5.4.1	Tangency portfolio with two risky assets	142
5.4.2	Combining the tangency portfolio with the risk-free asset	144
5.4.3	Effect of ρ_{12}	146
5.5	Risk-Efficient Portfolios with N Risky Assets*	146
5.5.1	Efficient-portfolio mathematics	146
5.5.2	The minimum variance portfolio	152
5.5.3	Selling short	154
5.5.4	Back to the math — Finding the tangency portfolio	156
5.5.5	Examples	157
5.6	Quadratic Programming*	160
5.7	Is the Theory Useful?	163
5.8	Utility Theory*	164
5.9	Summary	165

5.10	Bibliographic Notes	166
5.11	References	166
5.12	Problems	166
6	Regression	169
6.1	Introduction	169
6.1.1	Straight line regression	170
6.2	Least Squares Estimation	170
6.2.1	Estimation in straight line regression	171
6.2.2	Variance of $\hat{\beta}_1$	172
6.2.3	Estimation in multiple linear regression	174
6.3	Standard Errors, T -Values, and P -Values	174
6.4	Analysis Of Variance, R^2 , and F -Tests	177
6.4.1	AOV table	177
6.4.2	Sums of squares (SS) and R^2	177
6.4.3	Degrees of freedom (DF)	178
6.4.4	Mean sums of squares (MS) and testing	178
6.4.5	Adjusted R^2	179
6.4.6	Sequential and partial sums of squares	180
6.5	Regression Hedging*	181
6.6	Regression and Best Linear Prediction	183
6.7	Model Selection	183
6.8	Collinearity and Variance Inflation	187
6.9	Centering the Predictors	189
6.10	Nonlinear Regression	189
6.11	The General Regression Model	192
6.12	Troubleshooting	193
6.12.1	Influence diagnostics and residuals	194
6.12.2	Residual analysis	199
6.13	Transform-Both-Sides Regression*	206
6.13.1	How TBS works	210
6.13.2	Power transformations	214
6.14	The Geometry of Transformations*	214
6.15	Robust Regression*	216
6.16	Summary	217
6.17	Bibliographic Notes	219
6.18	References	220
6.19	Problems	220
7	The Capital Asset Pricing Model	225
7.1	Introduction to CAPM	225
7.2	The Capital Market Line (CML)	227
7.3	Betas and the Security Market Line	230
7.3.1	Examples of betas	231
7.3.2	Comparison of the CML with the SML	231

7.4	The Security Characteristic Line	232
7.4.1	Reducing unique risk by diversification	234
7.4.2	Can beta be negative?	235
7.4.3	Are the assumptions sensible?	235
7.5	Some More Portfolio Theory	236
7.5.1	Contributions to the market portfolio's risk	236
7.5.2	Derivation of the SML	237
7.6	Estimation of Beta and Testing the CAPM	238
7.6.1	Regression using returns instead of excess returns	241
7.6.2	Interpretation of alpha	241
7.7	Using CAPM in Portfolio Analysis	242
7.8	Factor Models	242
7.8.1	Estimating expectations and covariances of asset returns	244
7.8.2	Fama and French three-factor model	245
7.8.3	Cross-sectional factor models	246
7.9	An Interesting Question*	246
7.10	Is Beta Constant?*.	250
7.11	Summary	252
7.12	Bibliographic Notes	254
7.13	References	254
7.14	Problems	255
8	Options Pricing	257
8.1	Introduction	257
8.2	Call Options	258
8.3	The Law of One Price	259
8.3.1	Arbitrage	259
8.4	Time Value of Money and Present Value	260
8.5	Pricing Calls — A Simple Binomial Example	260
8.6	Two-Step Binomial Option Pricing	263
8.7	Arbitrage Pricing by Expectation	264
8.8	A General Binomial Tree Model	266
8.9	Martingales	268
8.9.1	Martingale or risk-neutral measure	269
8.9.2	The risk-neutral world	269
8.10	From Trees to Random Walks and Brownian Motion	270
8.10.1	Getting more realistic	270
8.10.2	A three-step binomial tree	270
8.10.3	More time steps	271
8.10.4	Properties of Brownian motion	273
8.11	Geometric Brownian Motion	273
8.12	Using the Black-Scholes Formula	276
8.12.1	How does the option price depend on the inputs?	276
8.12.2	Early exercise of calls is never optimal	277

8.12.3	Are there returns on nontrading days?	278
8.13	Implied Volatility	279
8.13.1	Volatility smiles and polynomial regression*	280
8.14	Puts	284
8.14.1	Pricing puts by binomial trees	284
8.14.2	Why are puts different from calls?	287
8.14.3	Put-call parity	287
8.15	The Evolution of Option Prices	288
8.16	Leverage of Options and Hedging	289
8.17	The Greeks	290
8.17.1	Delta and Gamma hedging	294
8.18	Intrinsic Value and Time Value*	295
8.19	Summary	295
8.20	Bibliographic Notes	297
8.21	References	298
8.22	Problems	298
9	Fixed Income Securities	301
9.1	Introduction	301
9.2	Zero-Coupon Bonds	302
9.2.1	Price and returns fluctuate with the interest rate	302
9.3	Coupon Bonds	304
9.3.1	A general formula	305
9.4	Yield to Maturity	305
9.4.1	General method for yield to maturity	306
9.4.2	MATLAB functions	307
9.4.3	Spot rates	308
9.5	Term Structure	309
9.5.1	Introduction: Interest rates depend upon maturity	309
9.5.2	Describing the term structure	310
9.6	Continuous Compounding	314
9.7	Continuous Forward Rates	315
9.8	Sensitivity of Price to Yield	316
9.8.1	Duration of a coupon bond	317
9.9	Estimation of a Continuous Forward Rate*	317
9.10	Summary	322
9.11	Bibliographic Notes	323
9.12	References	324
9.13	Problems	324
10	Resampling	327
10.1	Introduction	327
10.2	Confidence Intervals for the Mean	328
10.3	Resampling and Efficient Portfolios	332
10.3.1	The global asset allocation problem	332

10.3.2	Uncertainty about mean-variance efficient portfolios . . .	334
10.3.3	What if we knew the expected returns?	337
10.3.4	What if we knew the covariance matrix?	338
10.3.5	What if we had more data?	339
10.4	Bagging*	340
10.5	Summary	342
10.6	Bibliographic Notes	343
10.7	References	343
10.8	Problems	343
11	Value-At-Risk	345
11.1	The Need for Risk Management	345
11.2	VaR with One Asset	346
11.2.1	Nonparametric estimation of VaR	346
11.2.2	Parametric estimation of VaR	348
11.2.3	Estimation of VaR assuming Pareto tails*	349
11.2.4	Estimating the tail index*	350
11.2.5	Confidence intervals for VaR using resampling	353
11.2.6	VaR for a derivative	354
11.3	VaR for a Portfolio of Assets	355
11.3.1	Portfolios of stocks only	355
11.3.2	Portfolios of one stock and an option on that stock . .	356
11.3.3	Portfolios of one stock and an option on another stock	356
11.4	Choosing the Holding Period and Confidence	357
11.5	VaR and Risk Management	357
11.6	Summary	359
11.7	Bibliographic Notes	360
11.8	References	360
11.9	Problems	360
12	GARCH Models	363
12.1	Introduction	363
12.2	Modeling Conditional Means and Variances	364
12.3	ARCH(1) Processes	365
12.4	The AR(1)/ARCH(1) Model	368
12.5	ARCH(q) Models	370
12.6	GARCH(p, q) Models	370
12.7	GARCH Processes Have Heavy Tails	371
12.8	Comparison of ARMA and GARCH Processes	372
12.9	Fitting GARCH Models	372
12.10	I-GARCH Models	377
12.10.1	What does it mean to have an infinite variance?	379
12.11	GARCH-M Processes	381
12.12	E-GARCH	383
12.13	The GARCH Zoo*	386

12.14 Applications of GARCH in Finance	386
12.15 Pricing Options Under Generalized GARCH Processes*	387
12.16 Summary	391
12.17 Bibliographic Notes	392
12.18 References	392
12.19 Problems	393
13 Nonparametric Regression and Splines	397
13.1 Introduction	397
13.2 Choosing a Regression Method	400
13.2.1 Nonparametric regression	400
13.2.2 Linear	400
13.2.3 Nonlinear parametric regression	400
13.2.4 Comparison of linear and nonparametric regression	401
13.3 Linear Splines	405
13.3.1 Linear splines with one knot	405
13.3.2 Linear splines with many knots	406
13.4 Other Degree Splines	407
13.4.1 Quadratic splines	407
13.4.2 p th degree splines	408
13.5 Least Squares Estimation	409
13.6 Selecting the Spline Parameters	410
13.6.1 Estimating the volatility function	413
13.7 Additive Models*	415
13.8 Penalized Splines*	418
13.8.1 Penalizing the jumps at the knots	420
13.8.2 Cross-validation	421
13.8.3 The effective number of parameters	423
13.8.4 Generalized cross-validation	425
13.8.5 AIC	426
13.8.6 Penalized splines in MATLAB	427
13.9 Summary	431
13.10 Bibliographic Notes	431
13.11 References	431
13.12 Problems	432
14 Behavioral Finance	435
14.1 Introduction	435
14.2 Defense of the EMH	436
14.3 Challenges to the EMH	437
14.4 Can Arbitrageurs Save the Day?	438
14.5 What Do the Data Say?	439
14.5.1 Excess price volatility	439
14.5.2 The overreaction hypothesis	439
14.5.3 Reactions to earnings announcements	440

14.5.4 Counter-arguments to pricing anomalies	441
14.5.5 Reaction to non-news	442
14.6 Market Volatility and Irrational Exuberance	443
14.6.1 Best prediction	444
14.7 The Current Status of Classical Finance	445
14.8 Bibliographic Notes	445
14.9 References	446
14.10 Problems	447
Glossary	449
Index	461